

Lecture 8: K-Map to POS reductions

K-maps in higher dimensions

CSE 140: Components and Design Techniques for
Digital Systems

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Part I. Combinational Logic

1. Specification
2. Implementation

K-map:

Sum of products

Product of sums

Implicant: A **product term** that has non-empty intersection with on-set $\mathcal{F}$ and does not intersect with off-set $\mathcal{R}$.

Prime Implicant: An **implicant** that is not a proper subset of any other **implicant**.

Essential Prime Implicant: A prime **implicant** that has an element in on-set $\mathcal{F}$ but this element is not covered by any other prime **implicants**.

Implicate: A **sum term** that has non-empty intersection with off-set $\mathcal{R}$ and does not intersect with on-set $\mathcal{F}$.

Prime Implicate: An **implicate** that is not a proper subset of any other implicate.

Essential Prime Implicate: A prime **implicate** that has an element in off-set $\mathcal{R}$ but this element is not covered by any other prime **implicates**.

K-Map to Minimized Product of Sum

- Sometimes easier to reduce the K-map by considering the offset
 - Usually when number of zero outputs is less than number of outputs that evaluate to one OR offset is smaller than onset

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	1	1	1
11	0	1	1	1
10	1	1	1	1

Minimum Product of Sum: Ex1

Given $F(a,b,c) = \Sigma m(3, 5)$, $\mathcal{D} = \Sigma m(0, 4)$

ab \ c	00	01	11	10
0	⁰ X	² 0	⁶ 0	⁴ X
1	¹ 0	³ 1	⁷ 0	⁵ 1

Minimum Product of Sum: Ex 1

Given $F(a,b,c) = \sum m(3, 5)$, $\mathcal{D} = \sum m(0, 4)$

ab		00	01	11	10
c					
0	⁰ X	² 0	⁶ 0	⁴ X	
1	¹ 0	³ 1	⁷ 0	⁵ 1	

PI Q: The adjacent cells grouped in red minimize to the following sum term:

A. $a+b$

B. $(a+b)'$

C. $a'+b'$

Minimum Product of Sum: Ex1

Given $F(a,b,c) = \sum m(3, 5)$, $\mathcal{D} = \sum m(0, 4)$



ab		00	01	11	10
c	0	⁰ X	² 0	⁶ 0	⁴ X
	1	¹ 0	³ 1	⁷ 0	⁵ 1

Prime Implicates:

Essential Primes Implicates:

Min exp: $f(a,b,c) =$

Minimum Product of Sum: Ex 1

Given $F(a,b,c) = \sum m(3, 5)$, $\mathcal{D} = \sum m(0, 4)$



		ab			
		00	01	11	10
c	0	⁰ X	² 0	⁶ 0	⁴ X
	1	¹ 0	³ 1	⁷ 0	⁵ 1

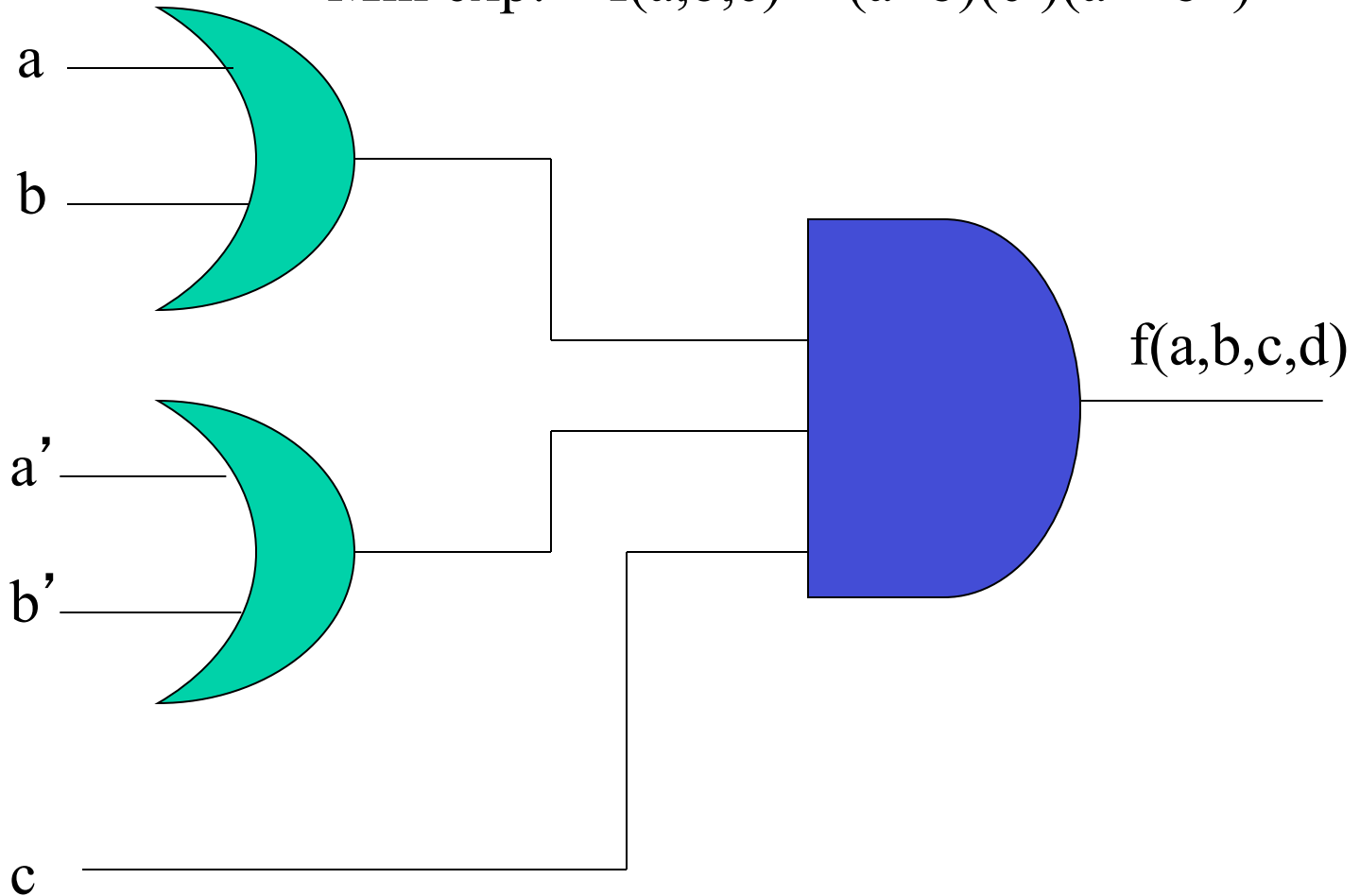
Prime Implicates: $\Pi M(0, 1)$, $\Pi M(0, 2, 4, 6)$, $\Pi M(6, 7)$

Essential Primes Implicates: $\Pi M(0, 1)$, $\Pi M(0, 2, 4, 6)$, $\Pi M(6, 7)$

Min exp: $f(a,b,c) = (a+b)(c')(a'+b')$

Corresponding Circuit

Min exp: $f(a,b,c) = (a+b)(c)(a' + b')$



Minimum product of sum: Ex 2

- Reduce the following to a POS form
- First find the essential prime implicants

ab		00	01	11	10
cd	00	0 1	4 0	12 0	8 1
	01	1 1	5 0	13 0	9 X
	11	3 0	7 0	15 0	11 0
	10	2 1	6 0	14 1	10 X

Minimum product of sum: Ex2

- Reduce the following to a POS form
- First find the essential prime implicants

ab		00	01	11	10
cd	00	0 1	4 0	12 0	8 1
	01	1 1	5 0	13 0	9 X
	11	3 0	7 0	15 0	11 0
	10	2 1	6 0	14 1	10 X

Minimum product of sum: Ex 2

- Reduce the following to a POS form
- First find the essential prime implicants

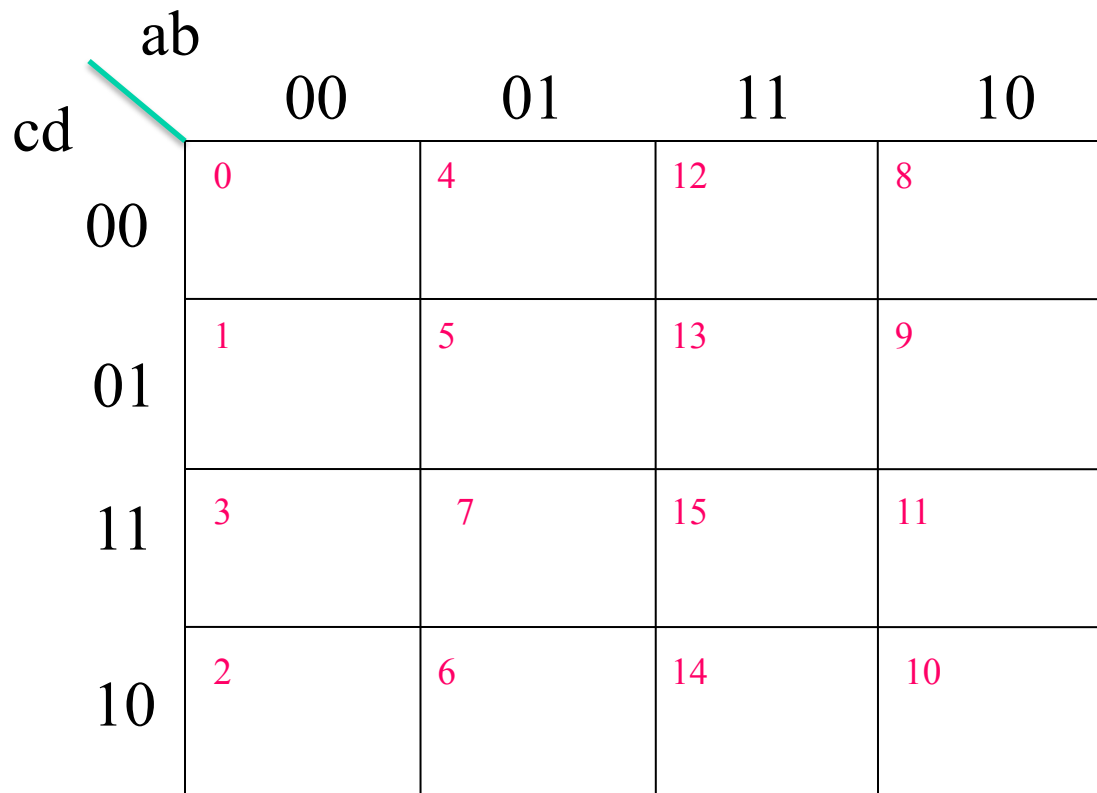
ab		00	01	11	10
cd	00	0 1	4 0	12 0	8 1
	01	1 1	5 0	13 0	9 X
	11	3 0	7 0	15 0	11 0
	10	2 1	6 0	14 1	10 X

Min product of sums: Ex3

Given $\mathcal{R}(a,b,c,d) = \Sigma m(3, 11, 12, 13, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m(4, 8, 10)$

K-map



ab	00	01	11	10
cd 00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Min product of sums: Ex3

Given $\mathcal{R}(a,b,c,d) = \Sigma m(3, 11, 12, 13, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m(4, 8, 10)$

		ab			
		00	01	11	10
cd	00	⁰ 1	⁴ X	¹² 0	⁸ X
	01	¹ 1	⁵ 1	¹³ 0	⁹ 1
	11	³ 0	⁷ 1	¹⁵ 1	¹¹ 0
	10	² 1	⁶ 1	¹⁴ 0	¹⁰ X
		a			

d

Prime Implicates: $\Pi M(3,11)$, $\Pi M(12,13)$, $\Pi M(10,11)$, $\Pi M(4,12)$, $\Pi M(8,10,12,14)$

PI Q: Which of the following is a non-essential prime implicate?

- A. $\Pi M(3,11)$
- B. $\Pi M(12,13)$
- C. $\Pi M(10,11)$
- D. $\Pi M(8,10,12,14)$

		ab			
		00	01	11	10
cd	00	⁰ 1	⁴ X	¹² 0	⁸ X
	01	¹ 1	⁵ 1	¹³ 0	⁹ 1
	11	³ 0	⁷ 1	¹⁵ 1	¹¹ 0
	10	² 1	⁶ 1	¹⁴ 0	¹⁰ X

d

a

(V) (25pts) (Karnaugh Map) Use Karnaugh map to simplify function $f(a,b,c) = \sum m(1,6) + \sum d(0,5)$. List **all possible** minimal **product of sums** expressions. Show the Boolean expressions. No need for the logic diagram.

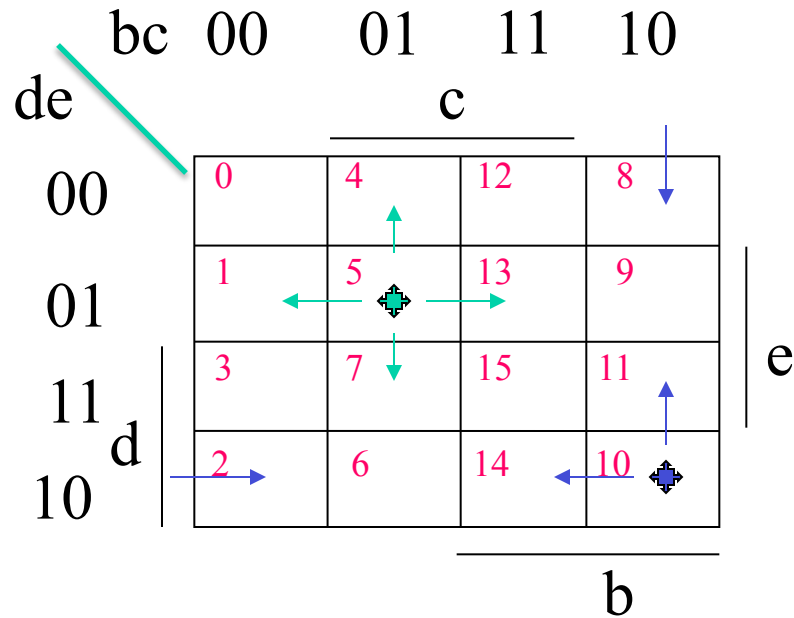
		ab			
		00	01	11	10
c	0	⁰ X	² 0	⁶ 1	⁴ 0
	1	¹ 1	³ 0	⁷ 0	⁵ X



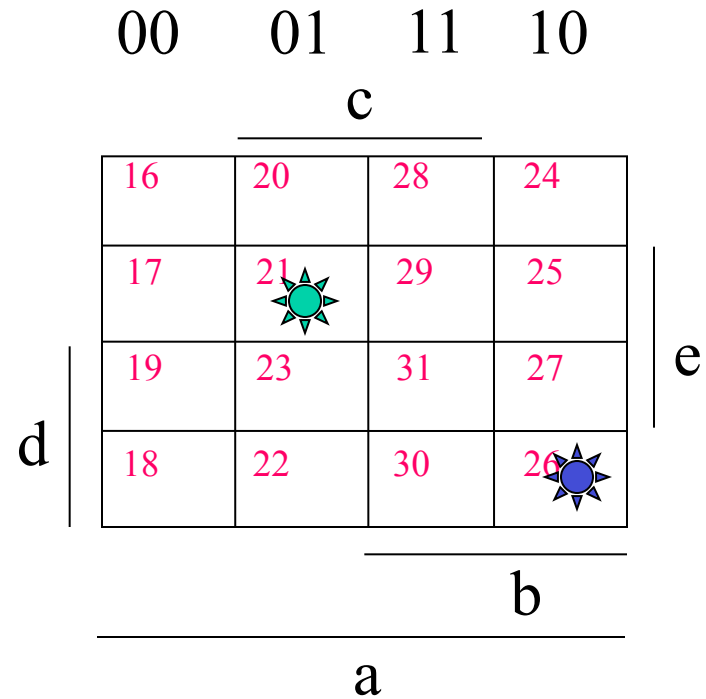
ab		00	01	11	10
c	0	⁰ X	² 0	⁶ 1	⁴ 0
	1	¹ 1	³ 0	⁷ 0	⁵ X

Five variable K-map

a=0



a=1



Neighbors of m_5 are: minterms 1, 4, 7, 13, and 21

Neighbors of m_{10} are: minterms 2, 8, 11, 14, and 26

Reading a Five variable K-map

a=0

		bc			
		00	01	11	10
		c			
de	00	0 1	4	12 1	8 1
	01	1	5 1	13 1	9
	11	3	7	15 1	11 1
	10	2 1	6	14 1	10 1
		b			

e

d

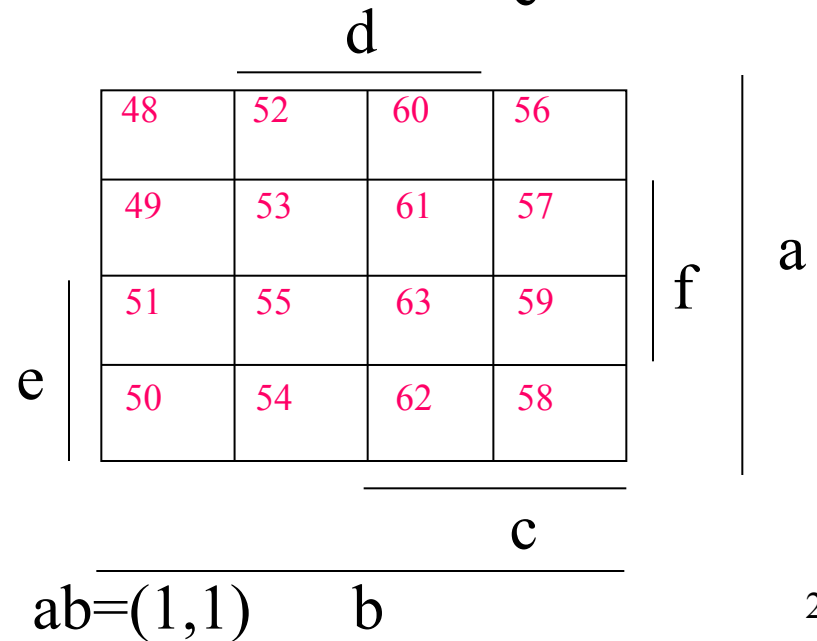
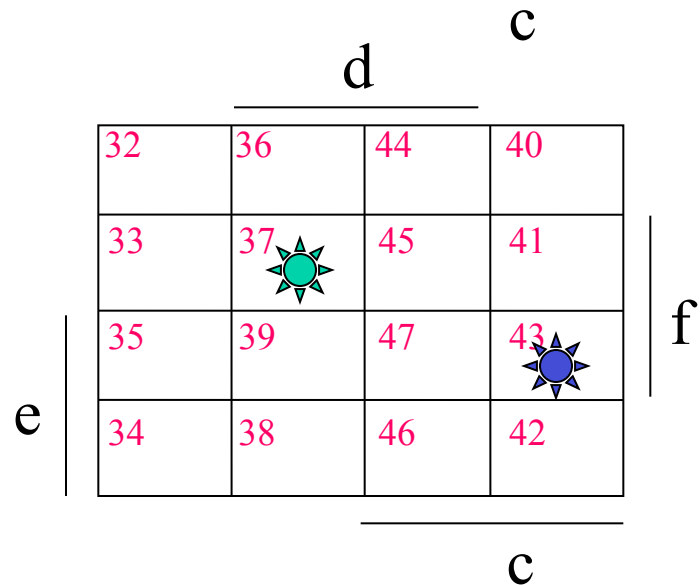
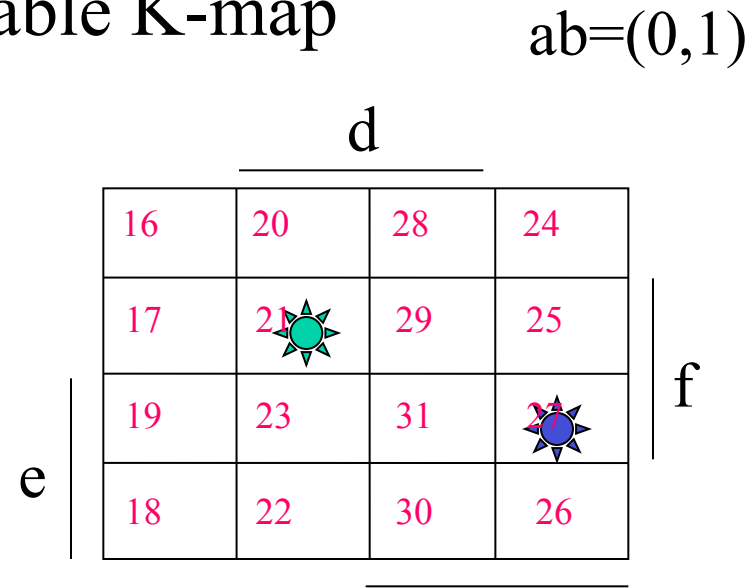
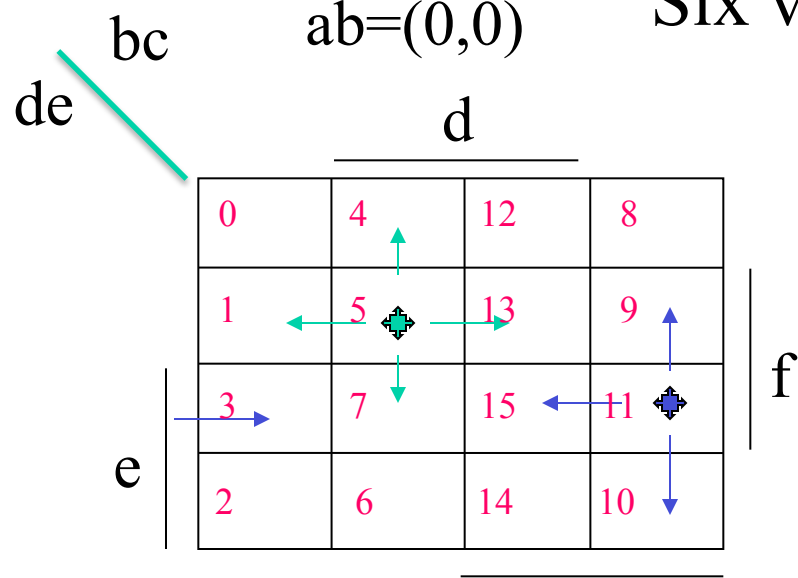


a=1

		bc			
		00	01	11	10
		c			
d	16	1	20	28	24 1
	17		21	29	25
	19		23 1	31 1	27 1
	18 1	22	30 1	26 1	
		b			
		a			

e

Six variable K-map



Reading

[Harris] Chapter 2, 2.7