

Lecture 16:
Sequential Networks –
Finite State Machines
Moore and Mealy (contd), circuit to FSM

CSE 140: Components and Design Techniques for
Digital Systems

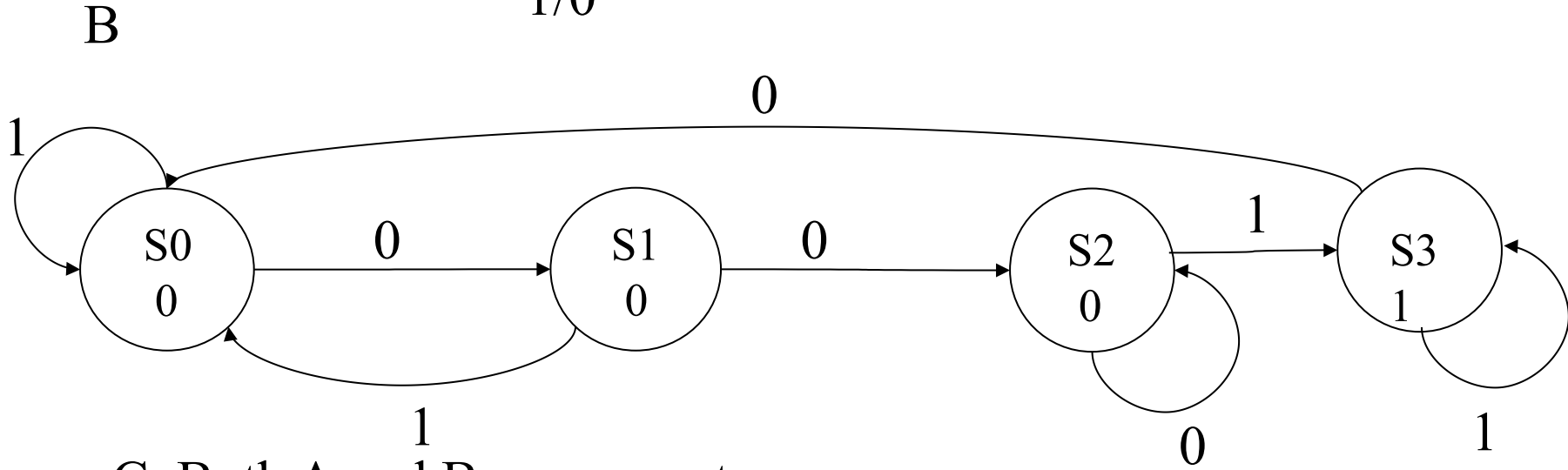
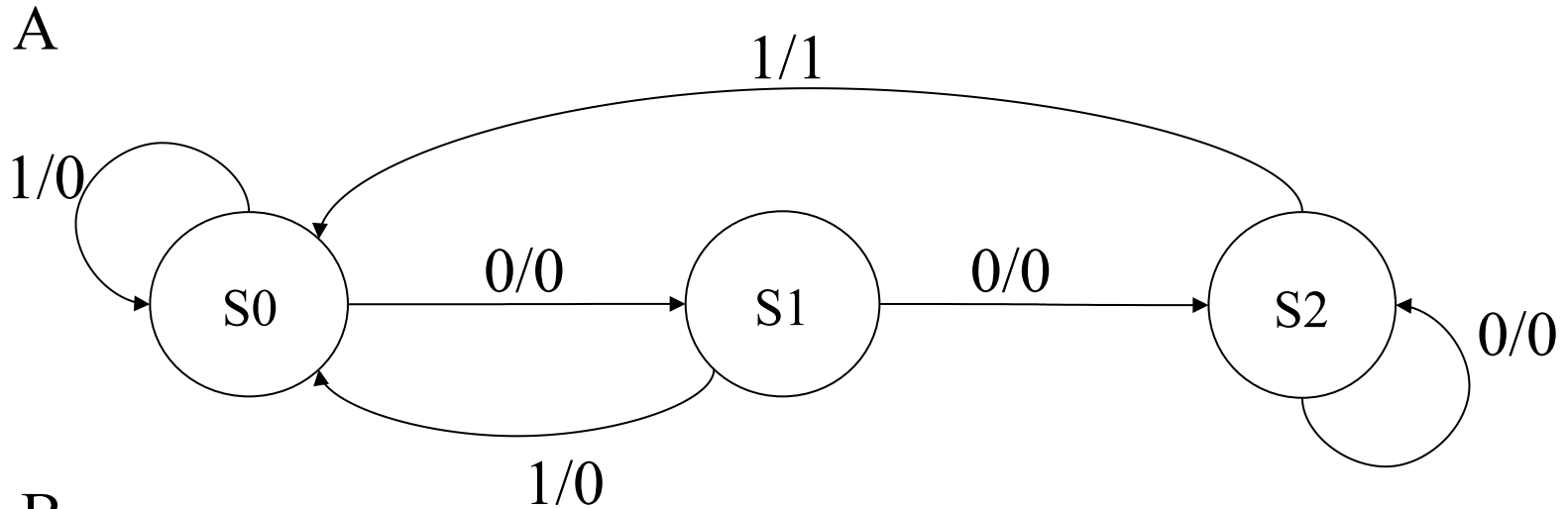
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Life on Mars?

Pattern Recognizer: A sequential machine has a binary input x (message from a remote robot on Mars). When the sequential machine receives the input sequence $x(t-2, t) = 001$, it means that the robot has detected life on Mars ☺ and the output $y(t) = 1$, otherwise $y(t) = 0$ (no life on Mars ☹).

Draw the Moore FSM for this problem.

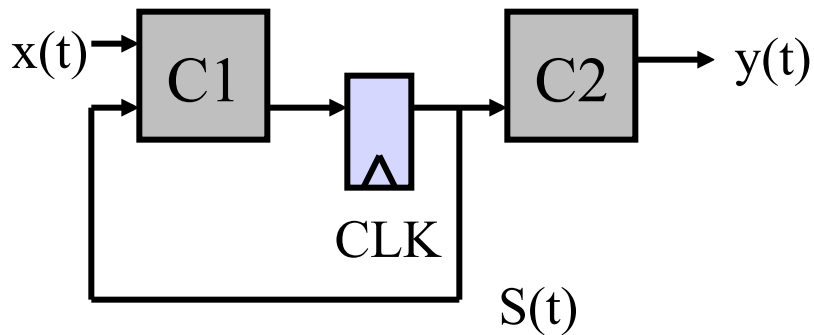
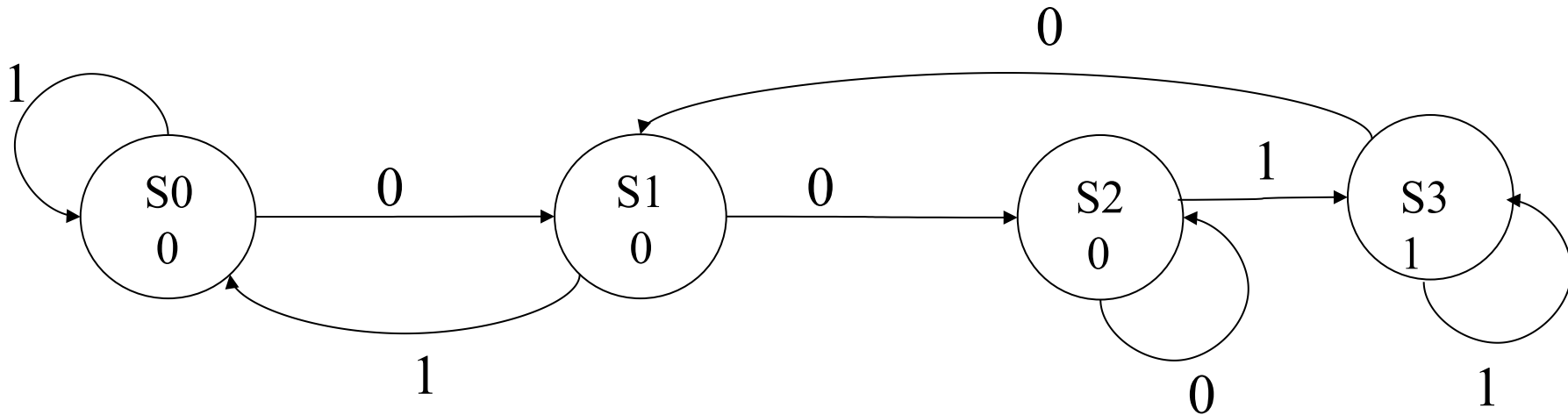
Q: Which of the following diagrams is a correct Moore solution to the '001' pattern recognizer?



C. Both A and B are correct

D. None of the above

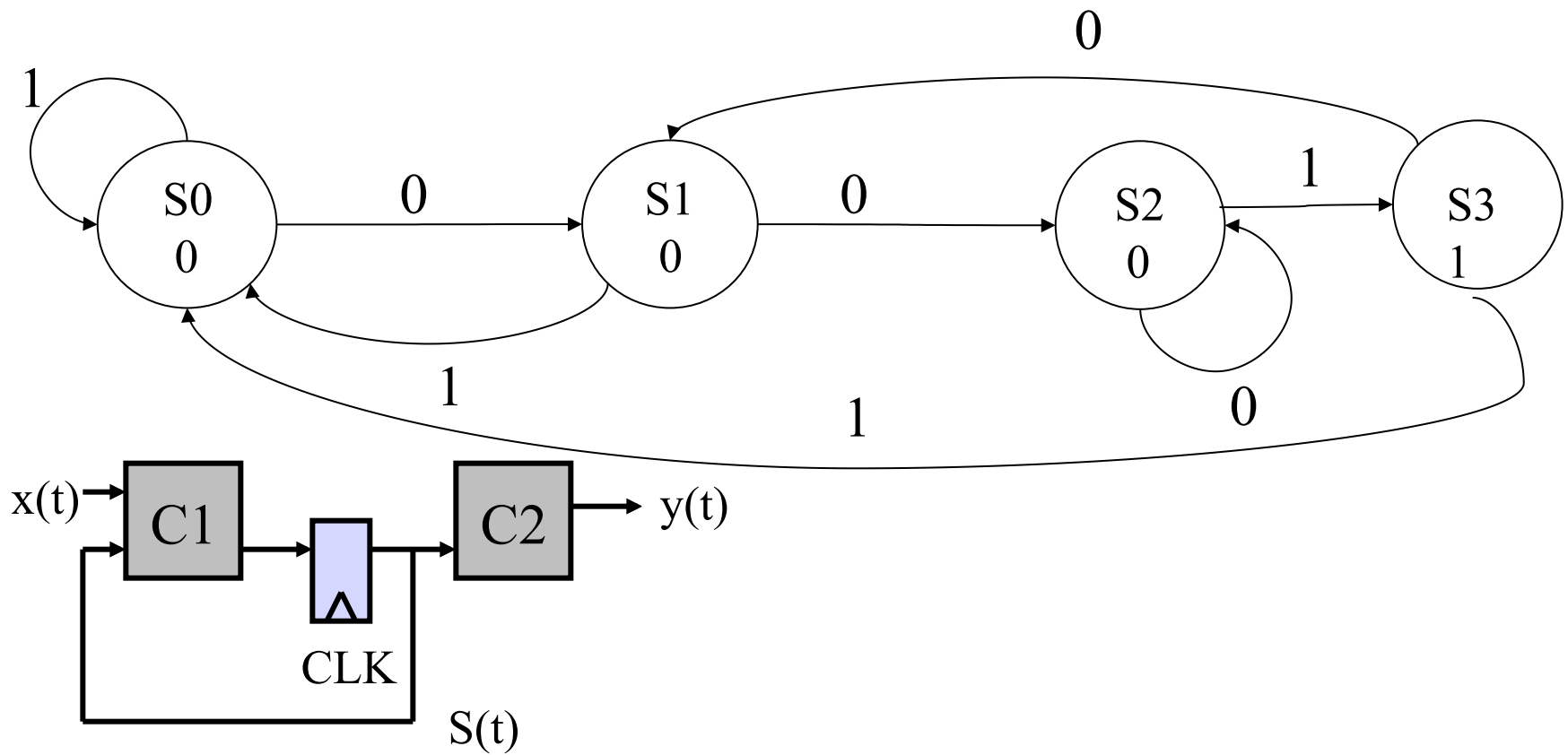
Pattern Recognizer '001'



Moore Machine

Q: To design C2 we need a state table showing?

- A. $x(t)$, $S(t)$ vs. $S(t+1)$
- B. $x(t)$, $S(t)$ vs. $y(t)$
- C. $S(t)$ vs. $S(t+1)$, $y(t)$
- D. $S(t)$ vs. $y(t)$
- E. None of the above



Moore Machine

$S(t) \backslash x$	0	1
S0	S1,0	S0,0
S1	S2,0	S0,0
S2	S2,0	S3,0
S3	S1,1	S0,1

State Assignment

S0: 00

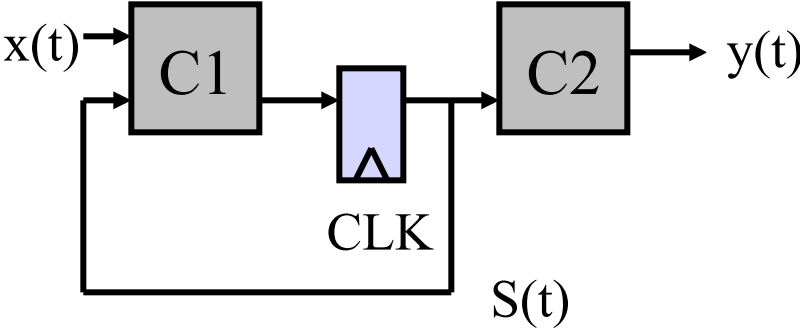
S1: 01

S2: 10

S3: 11

$Q_1(t+1)Q_0(t+1), y(t)$

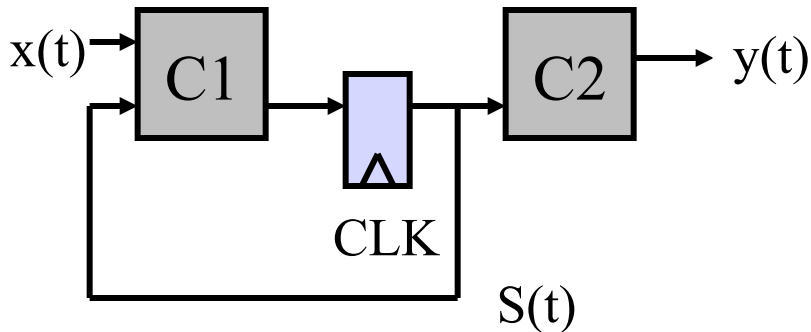
$Q_1Q_0 \backslash x$	0	1
00	01,0	00,0
01	10,0	00,0
10	10,0	11,0
11	01,1	00,1



Moore Machine

id	$Q_1(t)Q_0(t)x(t)$	$T_1(t)$	$T_0(t)$	$Q_1(t+1)$	$Q_0(t+1)$	$y(t)$
0	000	0		0	1	0
1	001	0		0	0	0
2	010	1		1	0	0
3	011	0		0	0	0
4	100	0		1	0	0
5	101	0		1	1	0
6	110	1		0	1	1
7	111	1		0	0	1

id	$Q_1(t)Q_0(t)x(t)$	$T_1(t)$	$T_0(t)$	$Q_1(t+1)$	$Q_0(t+1)$	$y(t)$
0	000	0	1	0	1	0
1	001	0	0	0	0	0
2	010	1	1	1	0	0
3	011	0	1	0	0	0
4	100	0	0	1	0	0
5	101	0	1	1	1	0
6	110	1	0	0	1	1
7	111	1	1	0	0	1



Moore Machine

$T_1(t):$

Q_0			
⁰ 0	² 1	⁶ 1	⁴ 0
¹ 0	³ 0	⁷ 1	⁵ 0

$x(t)$

Q_1

$T_0(t):$

Q_0			
⁰ 1	² 1	⁶ 0	⁴ 0
¹ 0	³ 1	⁷ 1	⁵ 1

$x(t)$

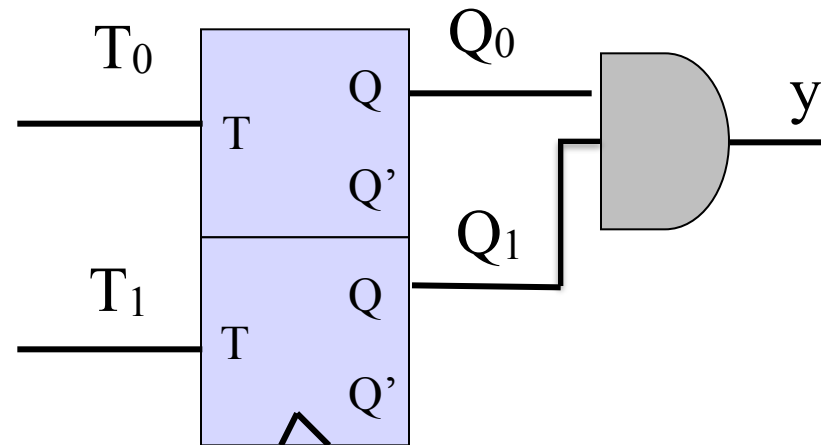
Q_1

$$T_1(t) = Q_0 x' + Q_1 Q_0$$

$$T_0(t) = Q_1 x + Q_1' x' + Q_1' Q_0$$

$$y = Q_1 Q_0$$

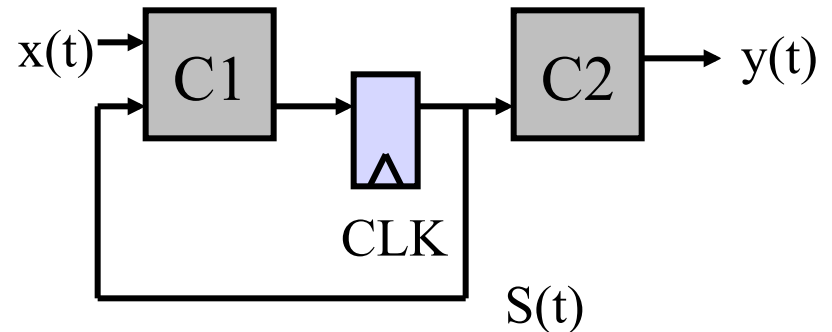
State Diagram => State Table => Excitation Table => **Circuit**



$$T_1(t) = Q_0 x' + Q_1 Q_0$$

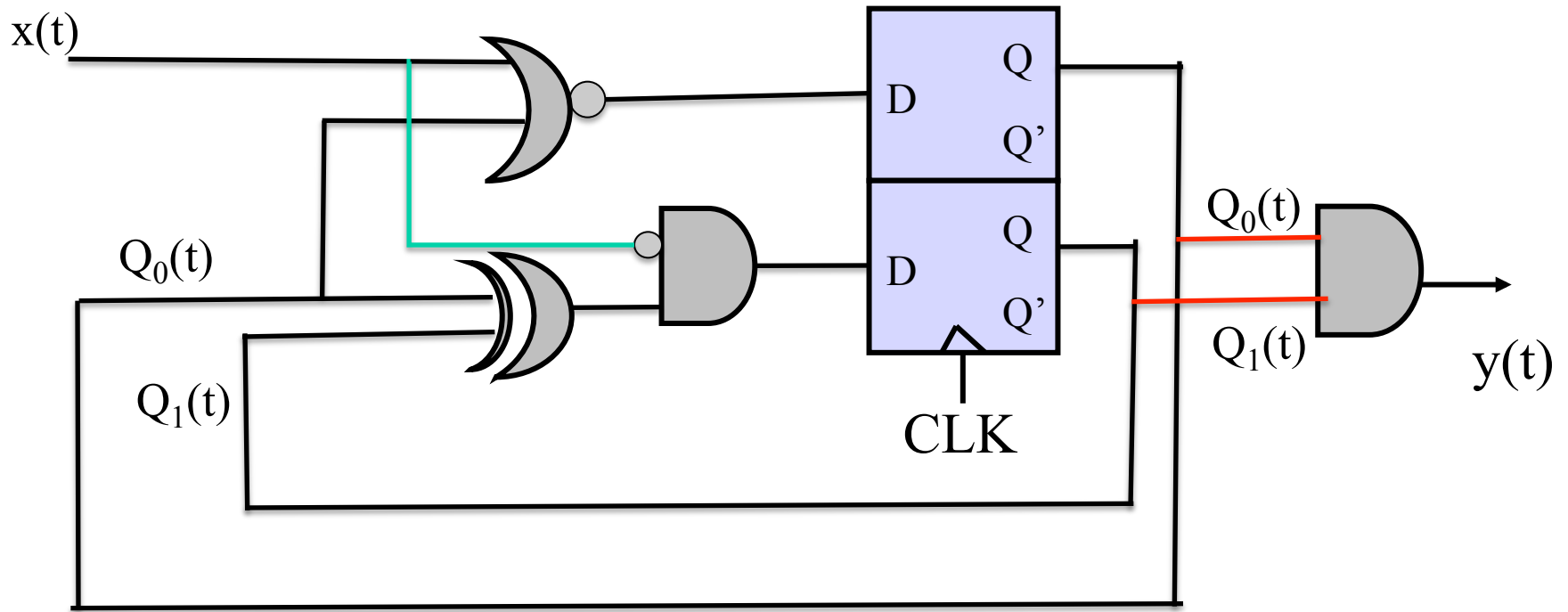
$$T_0(t) = Q_1 x + Q_1' x' + Q_1' Q_0$$

$$y = Q_1 Q_0$$

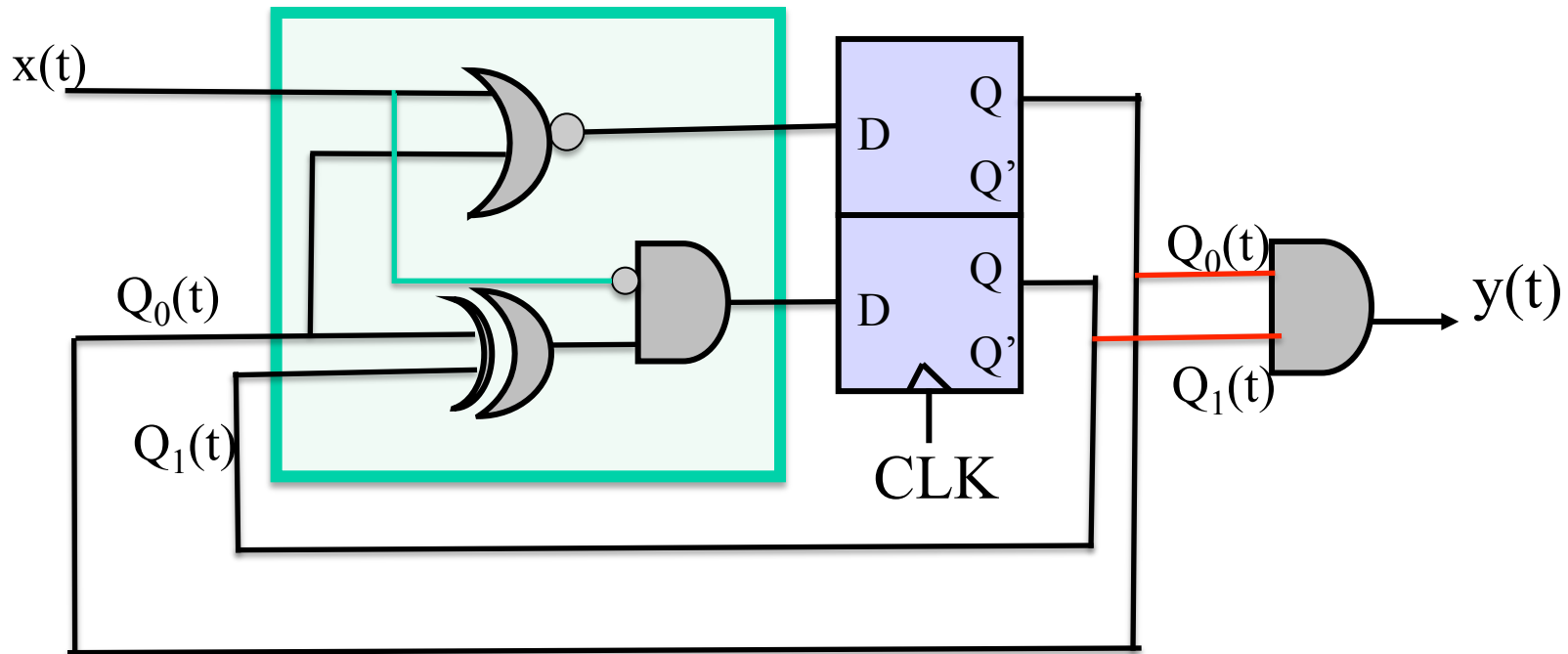


Moore Machine

FSM Specification



From circuit to FSM



$$y(t) = Q_1(t)Q_0(t)$$

$$Q_0(t+1) = D_0(t) = x(t)' Q_0(t)'$$

$$Q_1(t+1) = D_1(t) = x(t)'(Q_0(t) \oplus Q_1(t))$$

Netlist $\Leftrightarrow$ **State Table** $\Leftrightarrow$ State Diagram $\Leftrightarrow$ Input Output Relation

Characteristic Expression:

$$y(t) = Q_1(t)Q_0(t)$$

$$Q_0(t+1) = D_0(t) = x(t)' Q_0(t)'$$

$$Q_1(t+1) = D_1(t) = x(t)'(Q_0(t) \oplus Q_1(t))$$

State table

input PS		x=0	x=1
0	0		
0	1		
1	0		
1	1		

$Q_1(t) Q_0(t) \mid (Q_1(t+1) Q_0(t+1), y(t))$
Present State | **Next State, Output**

Netlist $\Leftrightarrow$ **State Table** $\Leftrightarrow$ State Diagram $\Leftrightarrow$ Input Output Relation

$$y(t) = Q_1(t)Q_0(t)$$

$$Q_0(t+1) = D_0(t) = x(t)' Q_0(t)'$$

$$Q_1(t+1) = D_1(t) = x(t)'(Q_0(t) \oplus Q_1(t))$$

State table

input			
PS		x=0	x=1
0 0		01, 0	00, 0
0 1		10, 0	00, 0
1 0		11, 0	00, 0
1 1		00, 1	00, 1

State Assignment

Let:

$$S_0 = 00$$

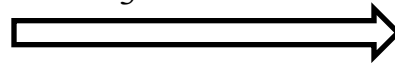
$$S_1 = 01$$

$$S_2 = 10$$

$$S_3 = 11$$

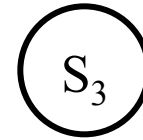
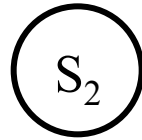
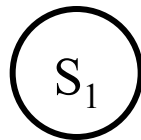
input			
PS		x=0	x=1
S_0		$S_1, 0$	$S_0, 0$
S_1		$S_2, 0$	$S_0, 0$
S_2		$S_3, 0$	$S_0, 0$
S_3		$S_0, 1$	$S_0, 1$

$Q_1(t) Q_0(t)$ | $Q_1(t+1) Q_0(t+1), y(t)$
 Present State | Next State, Output



Remake the state table using symbols instead of binary code , e.g. '00'

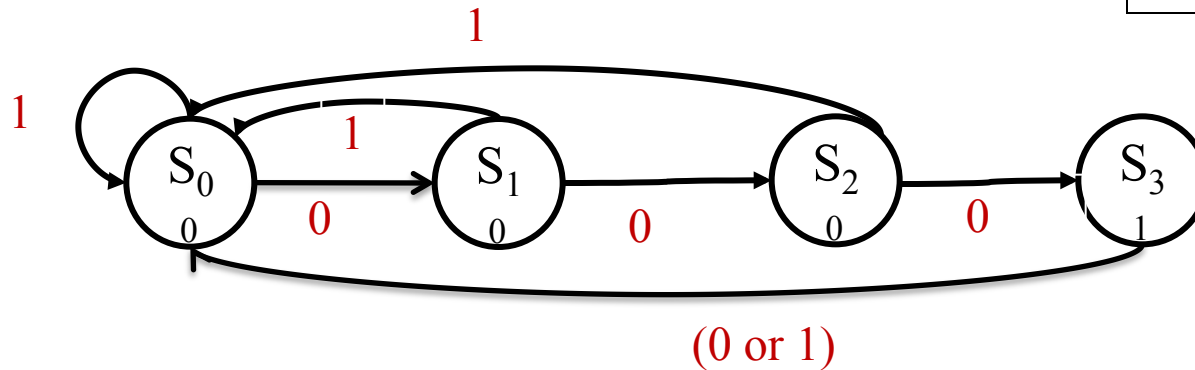
Netlist $\Leftrightarrow$ State Table $\Leftrightarrow$ State Diagram $\Leftrightarrow$ Input Output Relation



PS \ input	input	
	x=0	x=1
S_0	$S_1, 0$	$S_0, 0$
S_1	$S_2, 0$	$S_0, 0$
S_2	$S_3, 0$	$S_0, 0$
S_3	$S_0, 1$	$S_0, 1$

Netlist $\Leftrightarrow$ State Table $\Leftrightarrow$ State Diagram $\Leftrightarrow$ Input Output Relation

$\boxed{\mathbf{x/y}}$



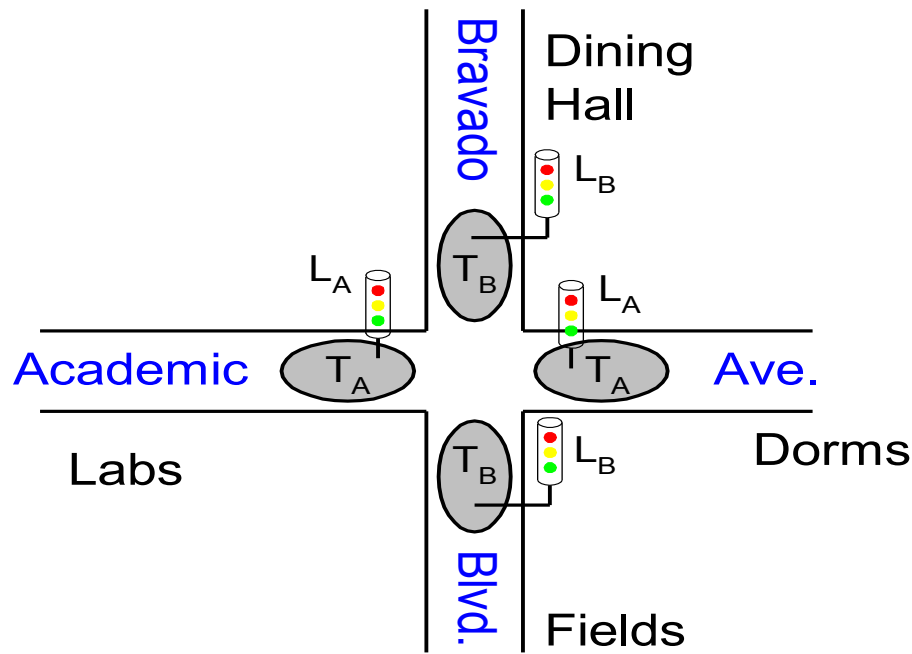
PS \ input	input	
	x=0	x=1
S ₀	S ₁ , 0	S ₀ , 0
S ₁	S ₂ , 0	S ₀ , 0
S ₂	S ₃ , 0	S ₀ , 0
S ₃	S ₀ , 1	S ₀ , 1

Example: Given inputs and initial state, derive output sequence

Time	0	1	2	3	4	5
Input	0	1	0	0	0	-
State	S0	S1	S0	S1	S2	S3
Output	0	0	0	0	0	1

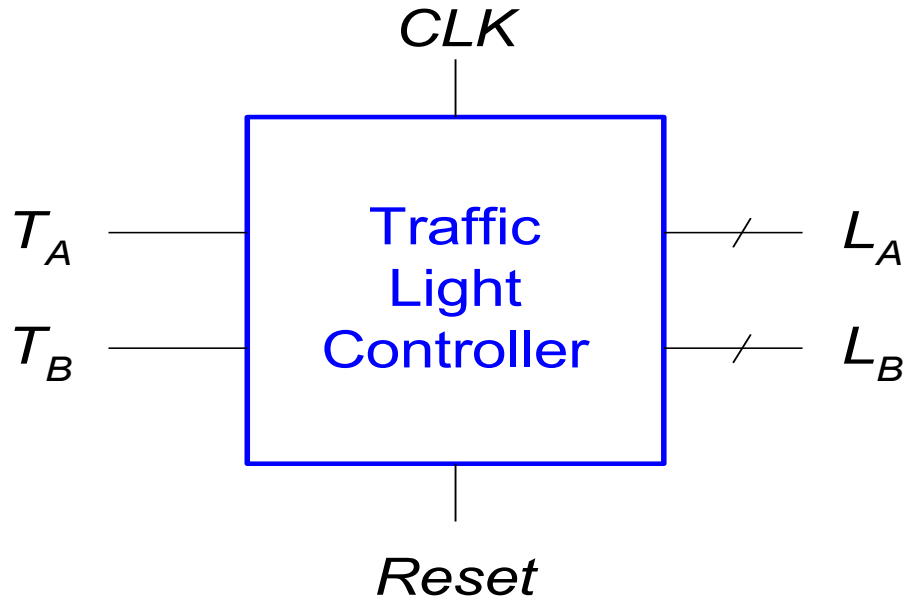
Finite State Machine Example

- Traffic light controller
 - Traffic sensors: T_A , T_B (TRUE when there's traffic)
 - Lights: L_A , L_B



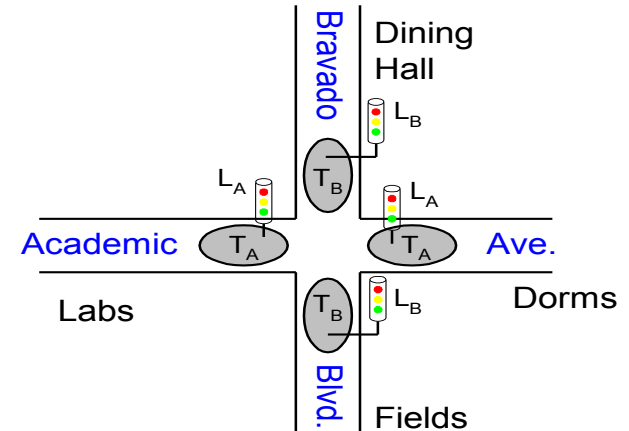
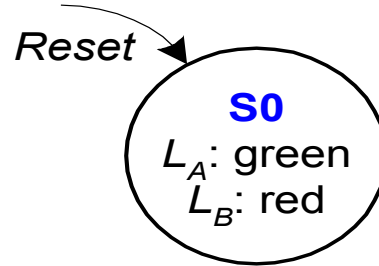
FSM Black Box

- Inputs: CLK , $Reset$, T_A , T_B
- Outputs: L_A , L_B



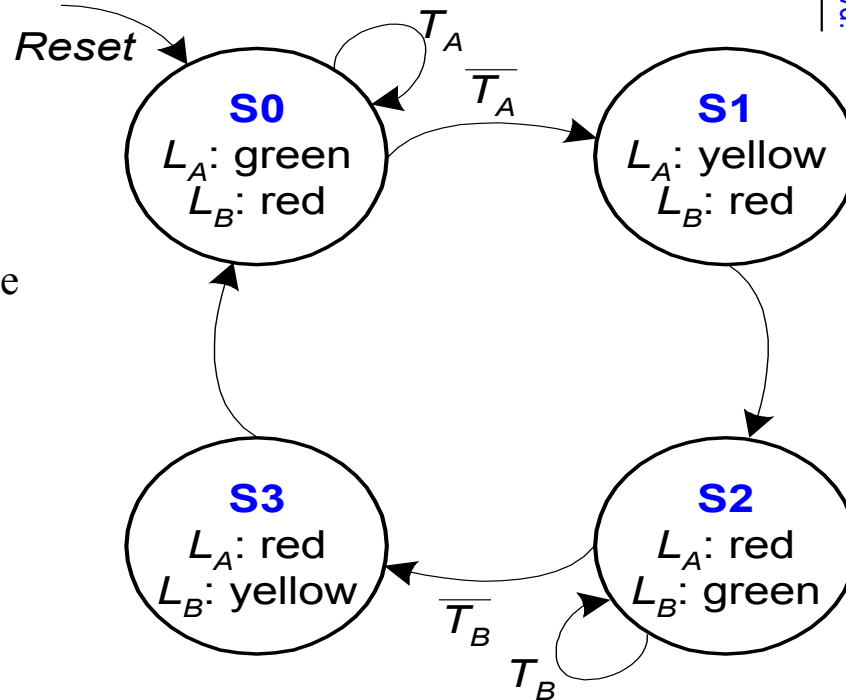
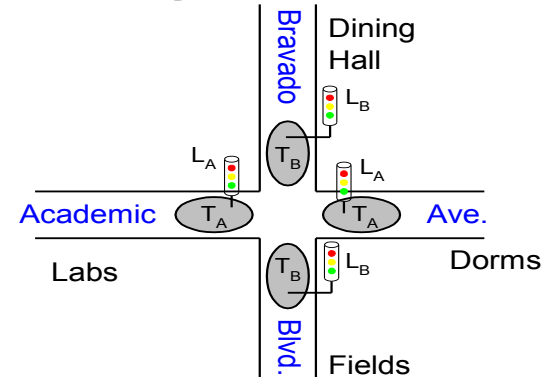
FSM State Transition Diagram

- Moore FSM: outputs labeled in each state
- States: Circles
- Transitions: Arcs



FSM State Transition Diagram

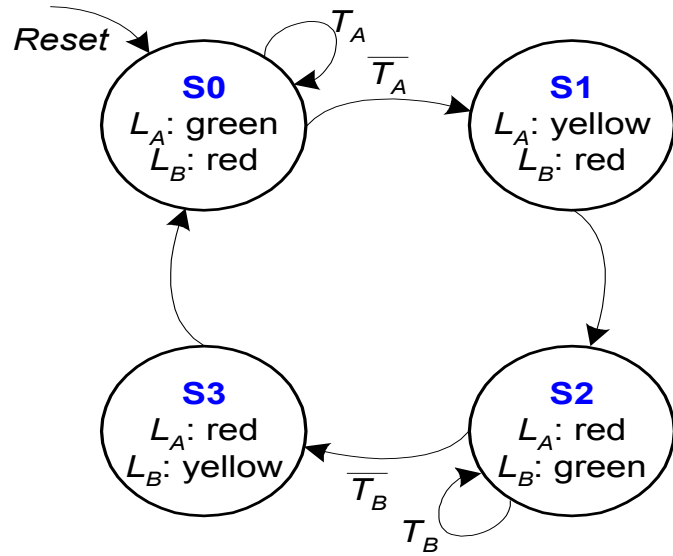
- Moore FSM: outputs labeled in each state



Which of the following is true about the controller?

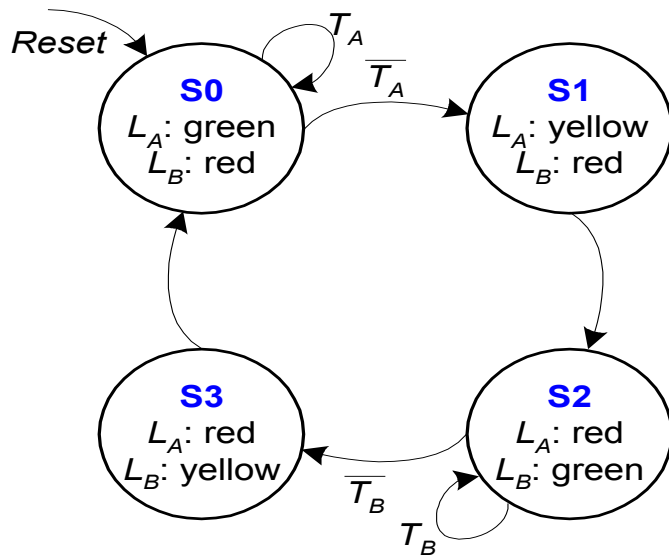
- The traffic light on Academic Ave (L_A) remains green as long as there is traffic on that street
- The traffic light on both avenues are green for exactly once clock cycle in every four clock cycles

FSM State Transition Table



PS	Inputs		NS	Output	
	T_A	T_B		L_A	L_B
S0	0	X	S1		
S0	1	X	S0		
S1	X	X	S2		
S2	X	0	S3		
S2	X	1	S2		
S3	X	X	S0		

FSM State Transition Table



PS	Inputs		NS	Output	
	T_A	T_B		L_A	L_B
S0	0	X	S1	green	red
S0	1	X	S0	green	red
S1	X	X	S2	yellow	red
S2	X	0	S3	red	green
S2	X	1	S2	red	green
S3	X	X	S0	red	yellow

State	Encoding
S0	00
S1	01
S2	10
S3	11

PS		Inputs		NS	
$Q_1(t)$	$Q_0(t)$	T_A	T_B	$Q_1(t+1)$	$Q_0(t+1)$
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

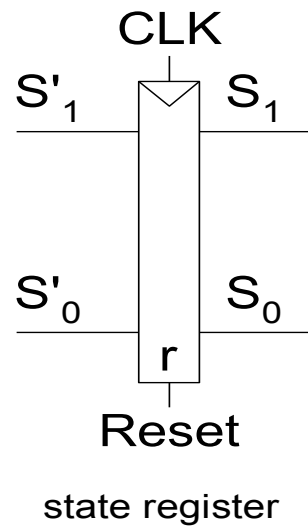
State Transition Table

PS		Inputs		NS	
$Q_1(t)$	$Q_0(t)$	T_A	T_B	$Q_1(t+1)$	$Q_0(t+1)$
0	0	0	X	0	1
0	0	1	X	0	0
0	1	X	X	1	0
1	0	X	0	1	1
1	0	X	1	1	0
1	1	X	X	0	0

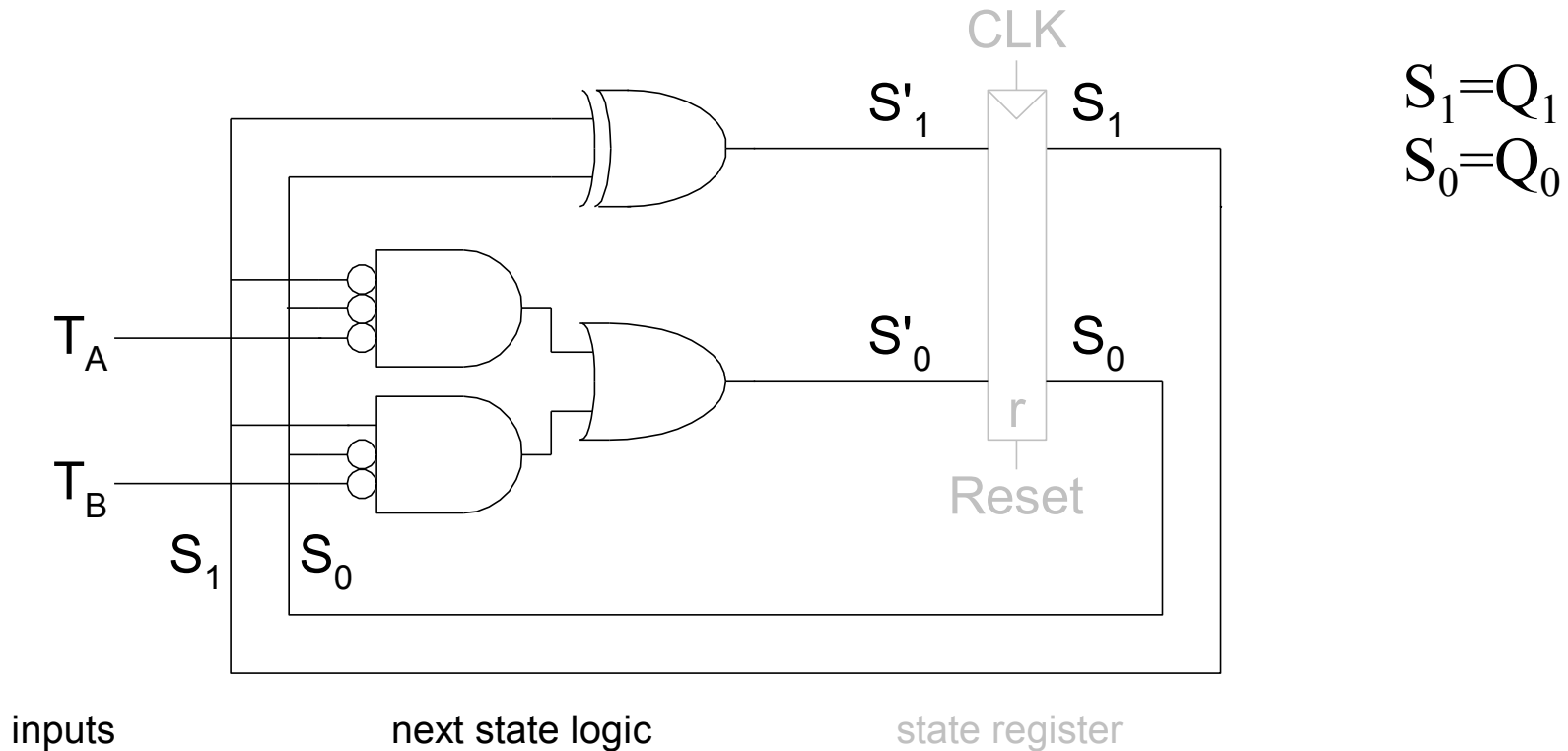
$$Q_1(t+1) = Q_1(t) \oplus Q_0(t)$$

$$Q_0(t+1) = Q_1'(t)Q_0'(t)T_A' + Q_1(t)Q_0'(t)T_B'$$

FSM Schematic: State Register



Logic Diagram



$$S_1 = Q_1$$

$$S_0 = Q_0$$

$$Q_1(t+1) = Q_1(t) \text{ AND } Q_0(t)$$

$$Q_0(t+1) = Q'_1(t)Q'_0(t)T'_A + Q_1(t)Q'_0(t)T'_B$$

FSM Output Table

PS	Inputs		NS	Output	
	T_A	T_B		L_A	L_B
S0	0	X	S1	green	red
S0	1	X	S0	green	red
S1	X	X	S2	yellow	red
S2	X	0	S3	red	green
S2	X	1	S2	red	green
S3	X	X	S0	red	yellow

Output	Encoding
green	00
yellow	01
red	10

PS		Outputs			
Q_1	Q_0	L_{A1}	L_{A0}	L_{B1}	L_{B0}
0	0	0	0	1	0
0	1	0	1	1	0
1	0	1	0	0	0
1	1	1	0	0	1

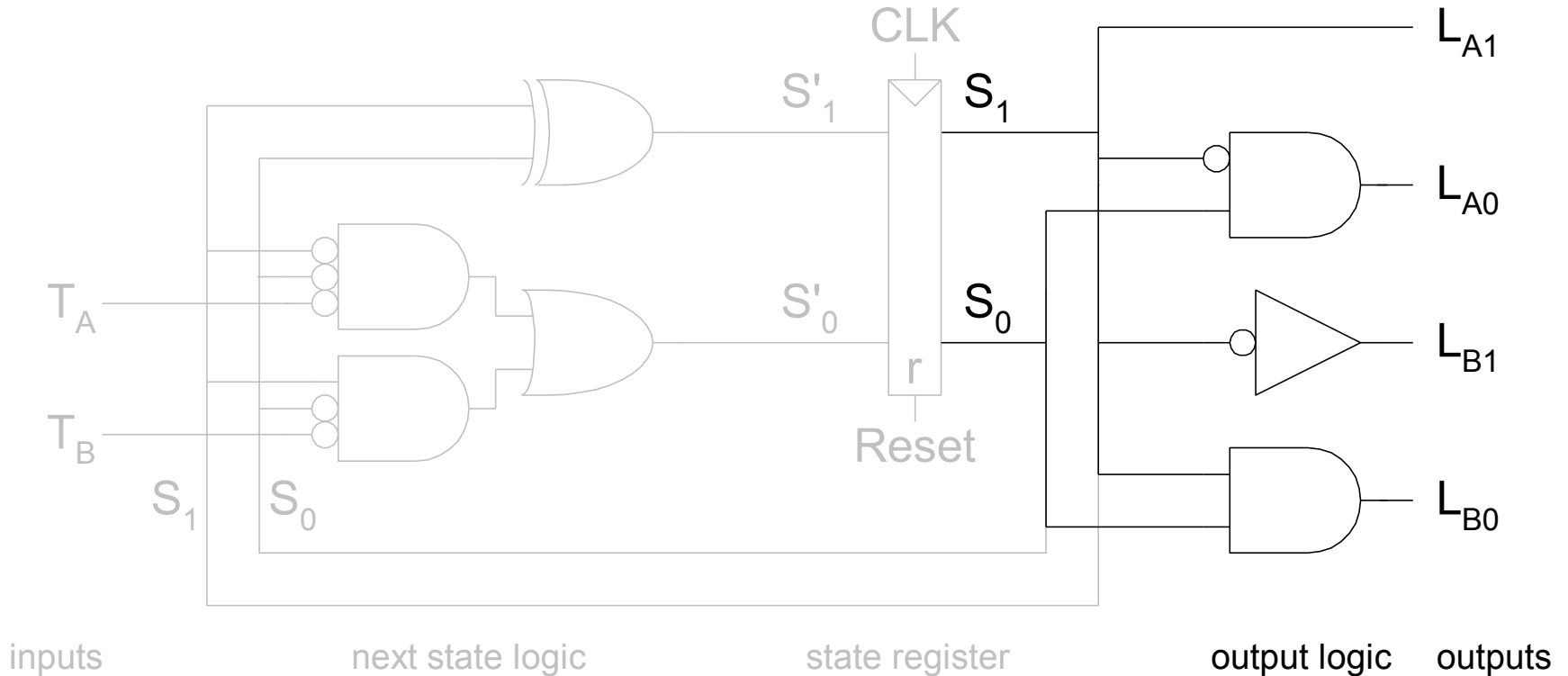
$$L_{A1} = Q_1$$

$$L_{A0} = Q_1' Q_0$$

$$L_{B1} = Q_1'$$

$$L_{B0} = Q_1 Q_0$$

FSM Schematic: Output Logic



$$L_{A1} = Q_1$$

$$L_{A0} = Q'_1 Q_0$$

$$L_{B1} = Q'_1$$

$$L_{B0} = Q_1 Q_0$$

Summary: Implementation

- Set up canonical form
 - Mealy or Moore machine
- Identify the next states
 - state diagram $\Rightarrow$ state table
 - state assignment
- Derive excitation table
 - Inputs of flip flops
- Design the combinational logic
 - don't care set utilization