

CSE 140

Discussion Section - Jan 16 '15

Topics

- Consensus Theorem
- Shannon's Expansion
- Truth Tables and Circuits

Consensus Theorem

In SOP form

$$AB + B'C + AC = AB + B'C$$

In POS form

$$(A+B)(B'+C)(A+C) = (A+B)(B'+C)$$

Proof of Consensus Theorem (POS)

$$(A + B)(B + C)(A + C') = (B + C)(A + C')$$

$$\begin{aligned}\text{LHS} &= (B + C)(C' + A)(A + B) \\ &= (B + C)(C' + A)(A + B + 0) \\ &= (B + C)(C' + A)(B + C.C' + A) \\ &= (B + C)(C' + A)(B + C + A)(B + C' + A) \\ &= [(B + C)(B + C + A)] [(C' + A)(B + C' + A)] \\ &= (B + C)(C' + A) = \text{RHS}\end{aligned}$$

Consensus Theorem (Visualize)

$$\textcolor{red}{A}.B + B'.\textcolor{blue}{C} + \textcolor{red}{A}.\textcolor{blue}{C} = A.B + B'.C$$

If $B = 0$, $\textcolor{blue}{LHS} = A.0 + 1.C + A.C = C + A.C = C$

$$\textcolor{brown}{RHS} = A.0 + 1.C = C = \textcolor{blue}{LHS}$$

If $B = 1$, $\textcolor{blue}{LHS} = A.1 + 0.C + A.C = A + A.C = A$

$$\textcolor{brown}{RHS} = A.1 + 0.C = A = \textcolor{blue}{LHS}$$

Examples

$$VX'Y + WXZ + VWYZ =$$

$$VX'Y + WXZ + VWYZ$$

$$= VX'Y + WXZ$$

$$(A + B + E') (E + F + G') (A + B + F + G') =$$

$$(A + B + E') (E + F + G') (A + B + F + G')$$

$$= (A + B + E') (E + F + G)$$

Shannon's Expansion

$$f(A, B, C) = A.f(1, B, C) + A'.f(0, B, C)$$

$$f(A, B, C) = (A' + f(1, B, C)) (A + f(0, B, C))$$

Note: Expansion can be done for any variable in the expression and can be repeated any number of times

Consensus using Shannon's Expansion

$$AB + B'C + AC = AB + B'C$$

$$\text{LHS} = AB + B'C + AC = f(A, B, C)$$

$$f(A, 0, C) = A.0 + 1.C + AC = C + AC = C$$

$$f(A, 1, C) = A.1 + 0.C + AC = A + AC = A$$

$$f(A, B, C) = B.f(A, 1, C) + B'.f(A, 0, C)$$

$$= BA + B'C = AB + B'C = \text{RHS}$$

Truth Tables and Circuits

Given a 3-bit input denoting day of week (000 = Sunday, 001 = Monday, ..., 110 = Saturday), construct a truth table and circuit to say if a given day is a weekend (1 if weekend, 0 otherwise)

Truth Tables and Circuits

Day	a2	a1	a0	$f(a2,a1,a0)$
Sunday	0	0	0	1
Monday	0	0	1	0
Tuesday	0	1	0	0
Wednesday	0	1	1	0
Thursday	1	0	0	0
Friday	1	0	1	0
Saturday	1	1	0	1

What about 1, 1, 1?

Truth Tables and Circuits

Day	a2	a1	a0	f(a2,a1,a0)
Sunday	0	0	0	1
Monday	0	0	1	0
Tuesday	0	1	0	0
Wednesday	0	1	1	0
Thursday	1	0	0	0
Friday	1	0	1	0
Saturday	1	1	0	1

What about 1, 1, 1? => Don't care

Truth Tables and Circuits

Day	a2	a1	a0	$f(a2,a1,a0)$
Sunday	0	0	0	1
Monday	0	0	1	0
Tuesday	0	1	0	0
Wednesday	0	1	1	0
Thursday	1	0	0	0
Friday	1	0	1	0
Saturday	1	1	0	1
Don't Care	1	1	1	X

Truth Tables and Circuits

Day	a2(a)	a1(b)	a0(c)	f(a,b,c)
Sunday	0	0	0	1
Monday	0	0	1	0
Tuesday	0	1	0	0
Wednesday	0	1	1	0
Thursday	1	0	0	0
Friday	1	0	1	0
Saturday	1	1	0	1
Don't Care	1	1	1	X

If $f(1, 1, 1) = 1$,

$$f(a, b, c) = a'b'c' + abc' + abc \\ = a'b'c' + ab$$

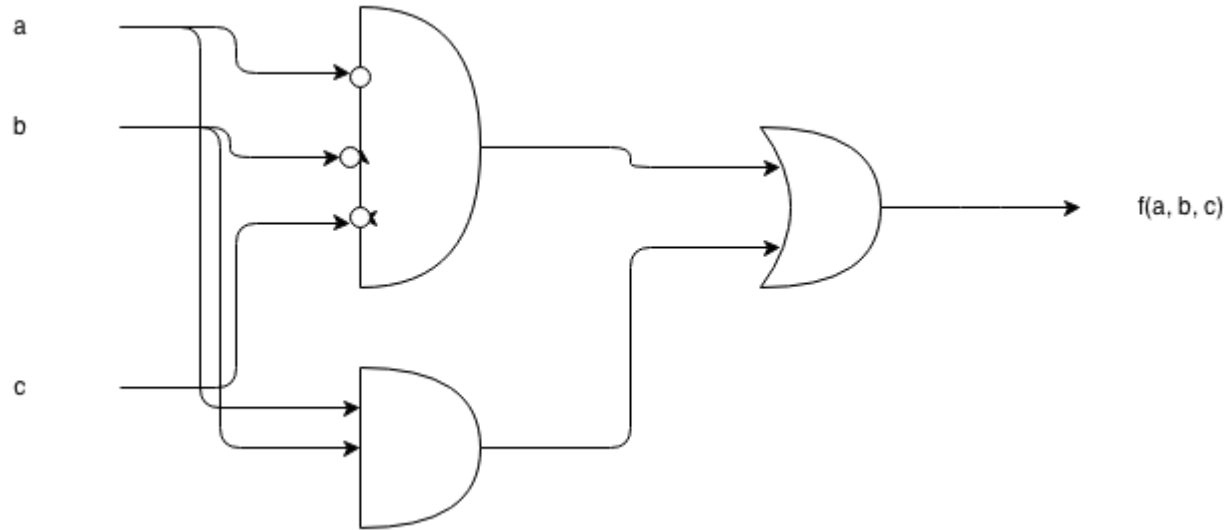
If $f(1, 1, 1) = 0$,

$$f(a, b, c) = a'b'c' + abc'$$

We can reduce number of literals with $f(1, 1, 1) = 1$

Truth Tables and Circuits

$$f(a, b, c) = a'b'c' + ab$$



Thank you!

Remember

- Post Questions on Piazza - Link on the course website
- HW 1 due next Wednesday, 21 Jan