

CSE140 Discussion

Logic Minimization with K maps

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Creating a Karnaugh map

- We can fill up a K map from the truth table of any boolean expression.
- Adjacent entries in K map differ only in one variable. Follows Gray code order.
- K map “wraps around”. Entries in first and last column/row can be combined. Imagine a cylinder rolled vertically and horizontally.

Four variable K map

- $F(a,b,c,d) = \sum m(0,2,3,5,6,7,8,9)$

a	b	c	d	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

cd\ab	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	0

$$F = a'c + a'bd + ab'c' + b'c'd'$$

Four variable K map

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0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

cd\ab	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	0

Alternate minimization

$$F = a'c + a'bd + ab'c' + a'b'd'$$

Four variable K map with Don't Cares

- $F(a,b,c,d) = \sum m(0,2,3,5,6,7,8,9) + \sum d(10,11,12,13,14,15)$

a	b	c	d	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

cd\ab	00	01	11	10
00	1	0	X	1
01	0	1	X	1
11	1	1	X	X
10	1	1	X	X

$$F = a + bd + b'd' + c$$

SOP and POS

- We can use K maps to find minimum POS and SOP expression
- If $(a'b'c' + a'b'c)$ are two adjacent min terms in K map, it reduces to $a'b'$
- If $(a+b+c)(a+b+c')$ are two adjacent max terms in K map, it reduces to $(a+b)$
- *Don't cares* can combine with 0s(for POS) and 1s(for SOP)

POS expression from K map

- $F(a,b,c,d) = \prod M(3,5,7,8,10,11,12,13)$

a	b	c	d	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

cd\ab	00	01	11	10
00	1	1	0	0
01	1	0	0	1
11	0	0	1	0
10	1	1	1	0

$$F = (a+c'+d')(b'+c+d')(a'+c+d)(a'+b+c')$$

POS expression from K map

- $F(a,b,c,d) = \prod M(3,5,7,8,10,11,12,13)$

a	b	c	d	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

cd\ab	00	01	11	10
00	1	1	0	0
01	1	0	0	1
11	0	0	1	0
10	1	1	1	0

Alternate minimization

$$F = (b+c'+d')(a+b'+d')(a'+b'+c)(a'+b+d)$$

There are multiple prime implicants but none of them are essential.

Questions?