

Homework #5

Due: Friday, March 6th, 2015, 11:59 PM

Submit your solutions as HW51.pdf, HW52.pdf, HW53.pdf, HW54.pdf using the bundleHW5 command on ieng6.

Problem 1

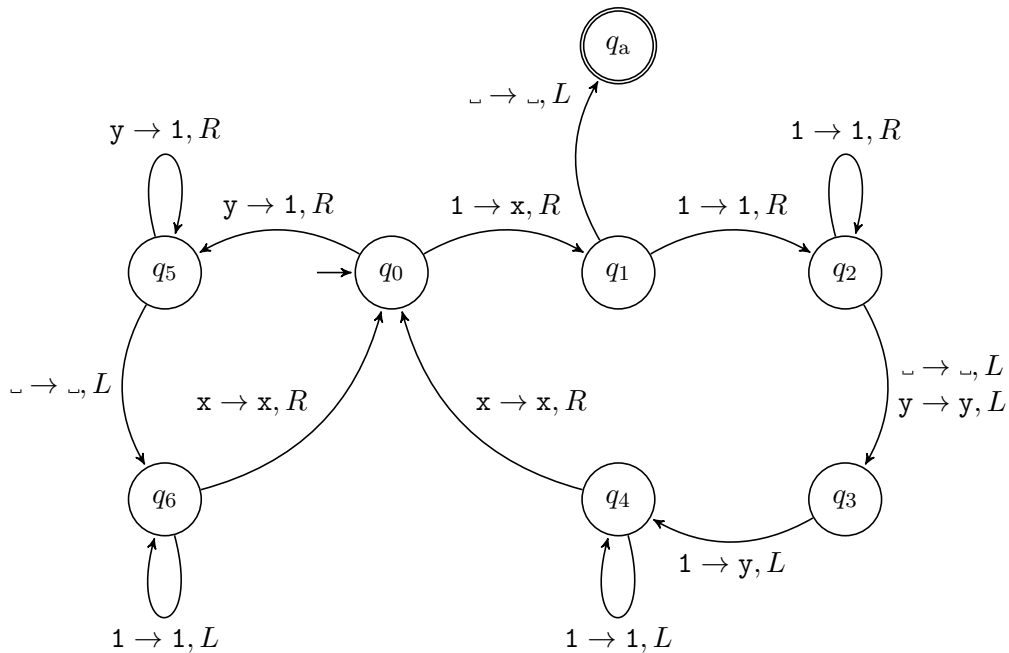
Show that the following language is not context-free:

$$L = \{a^i b^j c^k d^l \mid i, j, k, l \geq 0 \text{ and } i = k \text{ and } j = l\}$$

Here the alphabet is $\Sigma = \{a, b, c, d\}$.

Problem 2

Consider the following state diagram for a Turing machine with input alphabet $\Sigma = \{1\}$ and tape alphabet $\Gamma = \{1, x, y, \sqcup\}$. Any transition not drawn is to q_r .



What is the language of this Turing machine? Explain how the machine works.

Note: If you wish, you may use JFLAP to examine how the machine operates on particular inputs.

Problem 3

Sipser 3.14, “if” direction only: Show that if a language is R.E. (i.e., Turing-recognizable) then it can be recognized by a deterministic queue automaton, as defined in the problem.

You should begin by formally defining both the syntax and semantics of a queue automaton. For our purposes, the transition function of the queue automaton has type $\delta: Q \times \Sigma_\epsilon \times \Gamma_\epsilon \rightarrow Q \times \Gamma_\epsilon$, where Σ is the input alphabet (which includes the special symbol \sqcup that occurs only once at the end of input) and Γ is the queue alphabet.

Problem 4

The input to a Turing machine is always a string, but we will want to use Turing machines to reason not just about strings but about objects such as graphs, automata, grammars, and even other Turing machines. In order to do this, we will *encode* each object O that is the input to the Turing machine as a string $\langle O \rangle$ over the machine’s input alphabet Σ ; the input to the Turing machine will consist of the object in encoded (i.e., string) form.

Explain how you would encode an NFA $A = (\Sigma_A, Q_A, \delta_A, q_{0A}, F_A)$ as a string $\langle A \rangle$ suitable for input to a Turing machine. You can choose anything convenient for the Turing machine’s alphabet Σ .