

Quiz 5 Solutions

1 Show that every connected graph G has at least one vertex u such that $G - u$ is connected.

This problem is simpler version of problem 3 of HW 9. It makes use of the following facts:

Fact 1 Given any connected graph G , there is a minimal connected sub-graph, which is a tree. That is, for any connected graph on n vertices, we can obtain a new tree of same n vertices but fewer edges and without cycles.

Fact 2 For any tree, there are at least two degree one nodes.

So, given a connected graph G , using Fact 1, we can obtain a subtree T which is connected and has same set of vertices as G . Now using Fact 2, this tree must have at least two degree one vertices. Let one of them be u . Now, notice that deleting u does not disconnect T . Since T is a sub-graph of G , deleting u from G still keeps G connected. ■

2 Which of the following is a valid degree sequence for a graph. If degree sequence is not valid then give reasons else draw a graph with the degree sequence:

1. 0, 0, 0, 0, 3
2. 2, 3, 3, 4, 4, 5
3. 4, 4, 4, 4, 4, 4, 4

1. No. Sum of degrees is 3, which is odd. But Euler's formula requires sum of degrees to be an even number.
2. No. Same reason as before.
3. Yes. Figure 1 shows an example graph. ■

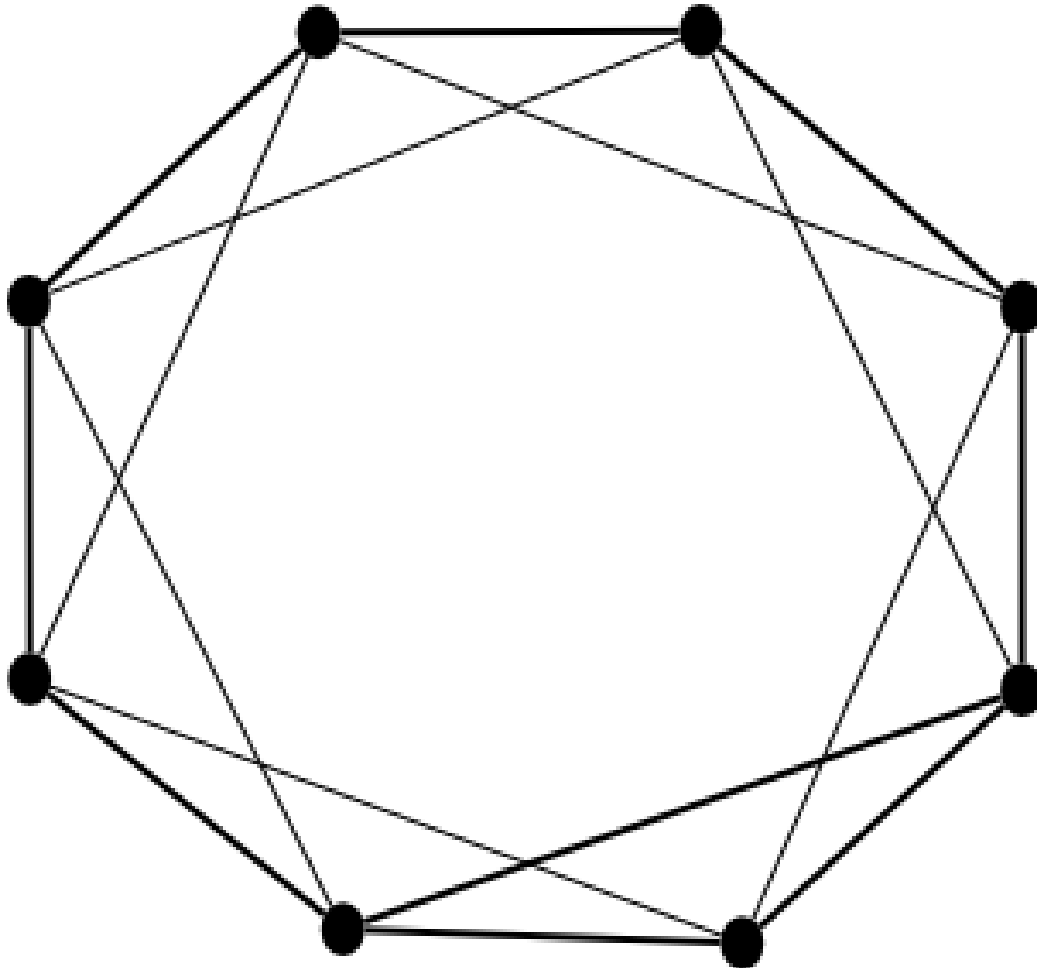


Figure 1: Figure for problem 2c