

2) If a dice is rolled, the output is either 1, 2, 3, 4, 5, 6 with same probability. What is the expected value of the output.

Let X denote the output of rolling a dice.
We want to compute $E(X)$.

$$P(X=i) = \frac{1}{6} \quad \text{for } 1 \leq i \leq 6.$$

$$E(X) = \sum_{i=1}^6 X \cdot P(X=i) \times i$$

$$= \sum_{i=1}^6 \frac{1 \times i}{6}$$

$$= \frac{1}{6} \left(\sum_{i=1}^6 i \right)$$

$$= \frac{1}{6} \times \frac{6 \times 7}{2}$$

$$= \underline{\underline{3.5}}$$

So, the expected value is 3.5

3. If a dice is biased in such a way that probability of getting '1' is double the probability of getting '2', while probability of getting other numbers, 3, 4, 5 & 6 is same as probability of getting 2. What is the expected value of a dice roll?

Let P_1 be probability of getting '1'.
 P_2 " " " " '2'

Then, probability of getting '3', '4', '5', '6', each is P_2 .

From question, we know $P_1 = 2P_2$. — (1)

Also, sum of all probabilities = 1.

$$\text{So, } P_1 + \underset{\substack{\uparrow \\ \text{For the numbers '2'-'6'}}}{5} \times P_2 = 1 \quad \text{--- (2)}$$

Combining (1) & (2), we get

$$\underline{P_2 = \frac{1}{7} \text{ \& } P_1 = \frac{2}{7}}$$

Let 'X' be the outcome of a single roll of this biased dice.

$$E(X) = \sum P(X=i) \times i$$

We know, $P(X=1) = p_1 = \frac{2}{7}$.

$$P(X=2) = P(X=3) = \dots = P(X=6) = p_2 = \frac{1}{7}$$

$$\text{So, } E(X) = 1 \times \frac{2}{7} + 2 \times \frac{1}{7} + 3 \times \frac{1}{7} + 4 \times \frac{1}{7} + 5 \times \frac{1}{7} + 6 \times \frac{1}{7}$$

$$= \frac{1}{7} (2 + 2 + 3 + 4 + 5 + 6)$$

$$= \frac{22}{7} \approx \underline{\underline{3.14}}$$

Answer is $\frac{22}{7} \approx 3.14$

4). A coin is biased and probability of HEAD is twice probability of TAIL. If coin is tossed 100 times, what is the expected number of heads.

Let P_H = Probability of Heads
 P_T = " " " Tails.

From question, $P_H = 2 P_T$.

Also, we know $P_H + P_T = 1$.

So, combining the two, we get $P_H = \frac{2}{3}$, $P_T = \frac{1}{3}$.

Let 'X' be the random variable which denotes the number of "Heads" we get on a ~~single~~
single coin toss. That is,

$$X = \begin{cases} 1 & \text{if we get "Heads"} \\ 0 & \text{if "Tails" in a single coin toss.} \end{cases}$$

$$E(X) = 1 \times P_H["\text{Heads"} \text{ in a single coin toss}] \\ + 0 \times P_T["\text{Tails"} \text{ in a single coin toss}]$$

$$= 1 \times \frac{2}{3} + 0 \times \frac{1}{3}$$

$$= \underline{\underline{\frac{2}{3}}}.$$

We want :

$$E[\text{"No. of 'Heads' in } \underline{100} \text{ coin tosses"}]$$

But, by linearity of expectations, ~~we~~ we know

$$E[\text{"No. of Heads in 100 coin toss"}]$$

$$= 100 \times E[\text{"No. of Heads in one coin toss"}]$$

$$= 100 \times E[X]$$

$$= 100 \times \frac{2}{3} = \frac{200}{3} \approx 66.67.$$

So, the final answer = 66.67