

CSE 21

Lecture 4: Applications of Graphs in solving problems

Reminder

- First quiz will be on Wednesday (15th January) in class.
- It is a paper quiz.
- Syllabus is all that you have done in CSE20 or Math15A. Mostly focused on induction.
- If you want you may bring a calculator

Representation of data

- Numbers - represented over any base. In particular one can represent a number over binary.
- Sets - binary vectors.
- Functions - Truth table.
- Relationship among elements - Graphs

Representation of relation among elements

- Vertices - set of elements.

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$$E = \{e_1, \dots, e_m\}$$

$$e_k = (v_i, v_j)$$

- Given the set of vertices and edges we have a graph

$$G = (V, E)$$

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- If there is an edge from vertex u to v we say v is a neighbor of u
- For an undirected graph the total number of u such that $(u, v) \in E$ is called degree of v .

Example of a graph

a

Vertices

b

c

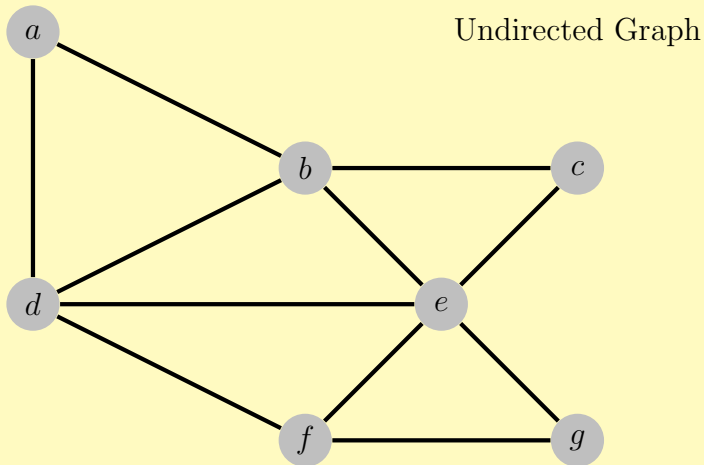
d

e

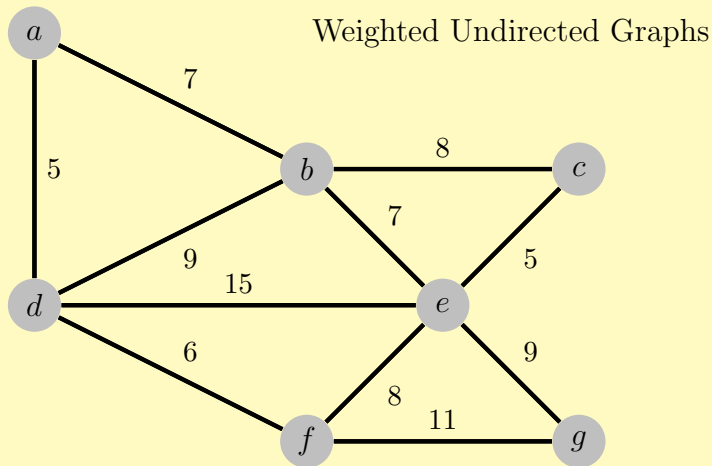
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g

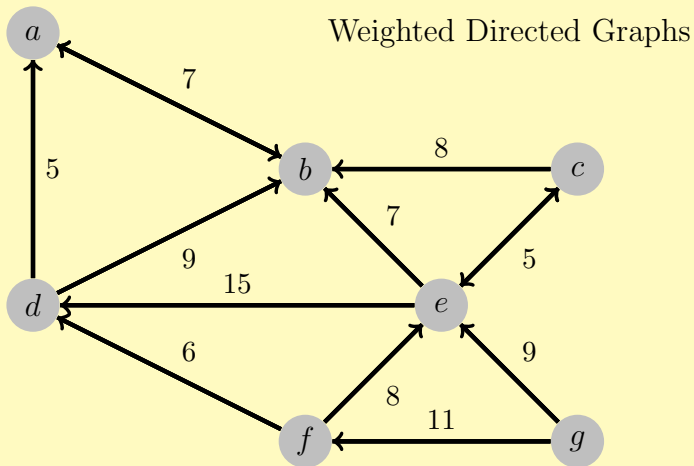
Example of a graph



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Advantages of a graph

- Mathematical way of expressing relations among objects.
- Very simple.
- Very general.

Graphs help us to visualize

In a room there are $2n$ people. Some of the people shake hand with each other in such a way that if persons A and B shake hand and persons B and C shake hand then person A and C does not shake hand.

What is the maximum number of handshakes possible in this case?

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Ans: n^2

Induction

To prove a statement, S_n for all n ,

- Prove the statement S_n for base cases (when $n = 0, 1, ..$
- Assuming the statement is true for case $n = k$ prove that the statement is true for $n = k + 1$.

Another example

Assignment 1, Problem 7:

Let $n > 1$ be an integer. In a football league there are n teams. Every two teams have played against each other exactly once, and in match no draw is allowed. Prove that it is possible to number the teams in such a way that team i beats $(i + 1)$ for $i = 1, 2, \dots, n - 1$.