

CSE 20: Assignment Set 3

1. Let r = “she registered to vote” and v = “she voted”. Write the following statement in symbolic form: She registered to vote but she did not vote.

“She registered to vote” AND “she did not vote”
 “She registered to vote ”AND NOT (“she voted”)
Answer: $r \wedge \sim v$

2. Make a truth table for $(p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$

I will show two ways to do this problem. The first way involves only filling a truth table. The second way involves algebraic manipulations before making the truth table.

Method 1

p	q	r	$\sim p$	$\sim p \vee q$	$p \vee (\sim p \vee q)$	$\sim r$	$q \wedge \sim r$	$\sim (q \wedge \sim r)$	$(p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$
T	T	T	F	T	T	F	F	T	T
T	T	F	F	T	T	T	T	F	F
T	F	T	F	F	T	F	F	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	F	F	T	T
F	T	F	T	T	T	T	T	F	F
F	F	T	T	T	T	F	F	T	T
F	F	F	T	T	T	T	F	T	T

Method 2

Note: This method uses the algebraic rules for boolean functions (Theorem 2 BF-6 of textbook). I tried to be as clear as possible by writing out every rule that was used in separate steps.

$$\begin{aligned}
 & (p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r) \\
 & ((p \vee \sim p) \vee q) \wedge \sim (q \wedge \sim r) && \text{associative rule} \\
 & (T \vee q) \wedge \sim (q \wedge \sim r) && \text{negation rule} \\
 & T \wedge \sim (q \wedge \sim r) && \text{bound rule} \\
 & T \wedge (\sim q \vee \sim \sim r) && \text{DeMorgan's rule} \\
 & T \wedge (\sim q \vee r) && \text{double negation} \\
 & \sim q \vee r && \text{bound rule}
 \end{aligned}$$

This means that the truth table for $(p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$ is equivalent to $\sim q \vee r$ so we can make a much simpler table.

p	q	r	$\sim q$	$\sim q \vee r$	$(p \vee (\sim p \vee q)) \wedge \sim (q \wedge \sim r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	T

3. Using DeMorgan's rule, state the negation of the statement: "The car is out of gas or the fuel line is plugged."

Let p = "the car is out of gas" and q = "the fuel line is plugged." Then, the statement $s = p \vee q$. The negation of s is $\sim s = \sim (p \vee q) = \sim p \wedge \sim q$ by DeMorgan's rule. So the negation reads "The car is not out of gas and the fuel line is not plugged."

4. A pair of numbers x and y satisfy a system of inequalities if

$$\begin{cases} 3 \leq x \leq 5 & \text{and} \\ |x - y| < 1. \end{cases}$$

What are the conditions under which x and y fail to satisfy this system?

This system can also be written as a conjunction of two statements. Let $p = 3 \leq x \leq 5$ and $q = |x - y| < 1$. Then $\sim (p \wedge q) = \sim p \vee \sim q$ by DeMorgan's rule. So system fails when $x < 3$ or $x > 5$ or $|x - 1| \geq 1$

5. Is the function $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q)$ equal to the function $p \vee q$? Why or why not?

Answer: No.

Like question 2, I will use two methods to answer this question. The first way is uses truth tables and the second way uses the algebraic rules.

Method 1

p	q	$\sim p$	$\sim p \vee q$	$\sim (\sim p \vee q)$	$p \wedge (\sim (\sim p \vee q))$	$p \wedge q$	$(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q)$	$p \vee q$
T	T	F	T	F	F	T	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	F	F	T
F	F	T	T	F	F	F	F	F

The last two columns of the table are not the same, and thus, the two statements are not equivalent.

Method 2

$$\begin{array}{ll}
(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) & \\
(p \wedge (\sim \sim p \wedge \sim q)) \vee (p \wedge q) & \text{DeMorgan's rule} \\
(p \wedge (p \wedge \sim q)) \vee (p \wedge q) & \text{double negation} \\
((p \wedge p) \wedge \sim q) \vee (p \wedge q) & \text{associative rule} \\
(p \wedge \sim q) \vee (p \wedge q) & \text{idempotent} \\
p \wedge (\sim q \vee q) & \text{distributive rule} \\
p \wedge \text{T} & \text{negation} \\
p & \text{bound rule}
\end{array}$$

6. If a and b are two positive integers then prove that $a^2 - 4b$ cannot be equal to 2.
(Hint: Prove using contradiction.)

Proof. Suppose for contradiction that a and b are positive integers and that $a^2 - 4b = 2$. Subtracting $4b$ from both sides gives us $a^2 = 2 + 4b = 2(1 + 4b)$. Since $1 + 4b$ is an integer, a^2 must be even, which means that a must be even. So, we can write $a = 2c$, where c is an integer. Substituting $2c$ for a , we get

$$\begin{aligned}
(2c)^2 - 4b &= 2 \\
4c^2 - 4b &= 2 \\
2c^2 - 2b &= 1 \\
2(c^2 - b) &= 1
\end{aligned}$$

The lefthand side of this equation is even, but the righthand side is odd, which is a contradiction. $\square$