

CSE 20

Lecture 5: Propositional Logic

Reminder

- First quiz will be on Friday (17th January) in class.
- It is a paper quiz.
- Syllabus is all that has been done till Wednesday.
- If you want you may bring a calculator

Equating Algebraic Equations

Is $(a^3 - b^3) = (a - b)(a^2 + 2ab + b^2)$?

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Is $(a^2 - b^2) = (a - b)(a + b)$?

How to check this equations?

Checking equalities in algebra

Is $(a^3 - b^3) = (a - b)(a^2 + 2ab + b^2)$?

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Checking equalities in algebra

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Is $(a^2 - b^2) = (a - b)(a + b)$?

- Try all possible values of x and y .

Checking equalities in algebra

Is $(a^3 - b^3) = (a - b)(a^2 + 2ab + b^2)$?

Is $(a^2 - b^2) = (a - b)(a + b)$?

- Try all possible values of x and y .
- Try to debate on it and decide.

Checking equalities in algebra

Is $(a^3 - b^3) = (a - b)(a^2 + 2ab + b^2)$?

Is $(a^2 - b^2) = (a - b)(a + b)$?

- Try all possible values of x and y .
- Try to debate on it and decide.
- Try to prove using algebraic deductions

Similar Logical Deductions

If you did not know the material earlier and you don't study hard then you will not get a A in this course.
Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

Is this sentence logically consistent?

Propositional logic

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- Every statement (proposition) is either TRUE or FALSE.

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- Statements are connected to each other by 5 connectives: AND, OR, NOT, IMPLIES and IFF.

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The above statement is either true or false.

Propositional Logic

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Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier = p

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier = p
 - you don't study hard

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier = p
 - you don't study hard = q

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier = p
 - you don't study hard = q
 - you will not get a A in this course

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier = p
 - you don't study hard = q
 - you will not get a A in this course = r

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier = p
 - you don't study hard = q
 - you will not get a A in this course = r
- What is “you knew this material earlier”?

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- Variable:
 - you did not know the material earlier = p
 - you don't study hard = q
 - you will not get a A in this course = r
- What is “you knew this material earlier”?
you knew this material earlier = $\neg p$

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- you did not know the material earlier = p

Writing a sentence as a proposition

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- you did not know the material earlier = p
- you don't study hard = q

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.
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- you don't study hard = q
- you will not get a A in this course = r

Writing a sentence as a proposition

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- you did not know the material earlier = p
- you don't study hard = q
- you will not get a A in this course = r
- you knew this material earlier = $\neg p$

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.
Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- you did not know the material earlier = p
- you don't study hard = q
- you will not get a A in this course = r
- you knew this material earlier = $\neg p$
- you studied hard = $\neg q$

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- you did not know the material earlier = p
- you don't study hard = q
- you will not get a A in this course = r
- you knew this material earlier = $\neg p$
- you studied hard = $\neg q$
- you get a A grade in this course = $\neg r$

Writing a sentence as a proposition

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

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Writing a sentence as a proposition

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Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

- you did not know the material earlier = p
- you don't study hard = q
- you will not get a A in this course = r

So the sentence is

$$((p \wedge q) \implies r) \implies (\neg r \implies (\neg p \wedge \neg q))$$

Writing a sentence as a proposition

The following sentence is logically correct:

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

Writing a sentence as a proposition

The following sentence is logically correct:

If you did not know the material earlier and you don't study hard then you will not get a A in this course.

Therefore if you get a A grade in this course then you knew this material earlier and you studied hard.

if and only if the following expression is correct/consistent:

$$((p \wedge q) \implies r) \implies (\neg r \implies (\neg p \wedge \neg q))$$

Checking for consistency

When can we say the following expression is correct / consistent?

$$((p \wedge q) \implies r) \implies (\neg r \implies (\neg p \wedge \neg q))$$

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When for all input variables the statement evaluate to **TRUE**.

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- Try all possible values of the input and see if it always evaluate to TRUE.

Checking for consistency

When can we say the following expression is correct / consistent?

$$((p \wedge q) \implies r) \implies (\neg r \implies (\neg p \wedge \neg q))$$

- Try all possible values of the input and see if it always evaluate to TRUE.

Questions

$$((p \wedge q) \implies r) \implies (\neg r \implies (\neg p \wedge \neg q))$$

- In the above expression how many variables are there?

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Ans: 3

- How many possible values of the input?

Questions

$$((p \wedge q) \implies r) \implies (\neg r \implies (\neg p \wedge \neg q))$$

- In the above expression how many variables are there?
Ans: 3
- How many possible values of the input? **Ans: $2^3 = 8$**

Checking for consistency

To check if an expression is correct / consistent we try all possible values of the input and see if it always evaluate to TRUE.

We create a table with all the possible input and the evaluations. We call it the **TRUTHTABLE** .

Truthtable

$$f = [(p \wedge q) \implies r] \implies (\neg r \implies (\neg p \wedge \neg q))$$

p	q	r	f
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
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$$f = [s \implies t]$$

p	q	r	$p \wedge q$	$g \rightarrow r$	$\neg p$	$\neg q$	$\neg r$	$p' \wedge q'$	$r' \rightarrow h$	f
			(g)	(s)	p'	q'	r'	(h)	(t)	
F	F	F								
F	F	T								
F	T	F								
F	T	T								
T	F	F								
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			(g)	(s)	p'	q'	r'	(h)	(t)	
F	F	F	F							
F	F	T	F							
F	T	F	F							
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			(g)	(s)	p'	q'	r'	(h)	(t)	
F	F	F	F	T						
F	F	T	F	T						
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			(g)	(s)	p'	q'	r'	(h)	(t)	
F	F	F	F	T	T					
F	F	T	F	T	T					
F	T	F	F	T	T					
F	T	T	F	T	T					
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F	F	F	F	T	T	T	T			
F	F	T	F	T	T	T	F			
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T	F	F	F	T	F	T	T			
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F	T	F	F	T	T	F	T	F		
F	T	T	F	T	T	F	F	F		
T	F	F	F	T	F	T	T	F		
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F	T	F	F	T	T	F	T	F	T	
F	T	T	F	T	T	F	F	F	T	
T	F	F	F	T	F	T	T	F	T	
T	F	T	F	T	F	T	F	F	T	
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			(g)	(s)	p'	q'	r'	(h)	(t)	
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F	T	F	F	T	T	F	T	F	T	T
F	T	T	F	T	T	F	F	F	T	T
T	F	F	F	T	F	T	T	F	T	T
T	F	T	F	T	F	T	F	F	T	T
T	T	F	T	F	F	F	T	F	T	T
T	T	T	T	T	F	F	F	F	T	T

Consistency/correctness of the expression

$$f = [(p \wedge q) \implies r] \implies (\neg r \implies (\neg p \wedge \neg q))$$

Since the expression does not evaluate to true always so the expression is not correct.