

CSE 20

Lecture 4: Introduction to Boolean algebra

Reminder

- First quiz will be on Friday (17th January) in class.
- It is a paper quiz.
- Syllabus is all that has been done till Wednesday.
- If you want you may bring a calculator

Base b representation

Base b representation

Digits: $0, 1, \dots, b - 1$

Base b representation

Digits: $0, 1, \dots, b - 1$

Represented as $[x]_b$. (Like $[22001]_3$)

Base b representation

Digits: $0, 1, \dots, b - 1$

Represented as $[x]_b$. (Like $[22001]_3$)

Base b representation of a number x is the unique way of writing

$$x = x_0 * b^0 + x_1 * b^1 + \dots + x_k * b^k,$$

where $x_0, x_1, \dots, x_k \in \{0, 1, \dots, (b - 1)\}$

Algorithm for finding representation in base b

We can find a representation in base b using **GREEDY METHOD**.

Unique representation in base “b”

Can an integer be written in base b in two different ways?

Answer may be obvious but we need to prove it mathematically.

Mathematical formulation of the unique representation in base “ b ” problem

Let N be a number that be write in base b .

Mathematical formulation of the unique representation in base “b” problem

Let N be a number that be write in base b .

Let there be two different representation in base b :

$$N = x_0 * b^0 + x_1 * b^1 + \cdots + x_k * b^k,$$

Mathematical formulation of the unique representation in base “b” problem

Let N be a number that be write in base b .

Let there be two different representation in base b :

$$N = x_0 * b^0 + x_1 * b^1 + \cdots + x_k * b^k,$$

$$N = y_0 * b^0 + y_1 * b^1 + \cdots + y_k * b^k.$$

Is it possible that there exists $(\exists) i$ such that $x_i \neq y_i$?

Proof style

- We prove by **contradiction** .

Proof style

- We prove by **contradiction** .
- We assume that a number can be written in two different ways

Proof style

- We prove by **contradiction** .
- We assume that a number can be written in two different ways
- Then using this assumption we conclude that something seriously wrong happens, like

$$2^\ell < 2^\ell$$

Proof style

- We prove by **contradiction** .
- We assume that a number can be written in two different ways
- Then using this assumption we conclude that something seriously wrong happens, like

$$2^\ell < 2^\ell$$

- So we conclude that the original assumption was wrong.

Mathematical logic

Mathematical logic

- Every statement (proposition) is either TRUE or FALSE.

Mathematical logic

- Every statement (proposition) is either TRUE or FALSE.
- A statement can have an unspecified term, called variable.
- Statements are connected to each other by 5 connectives: AND, OR, NOT, IMPLIES and IFF.

The IMPLIES ($\implies$)

p	q	$p \implies q$
F	F	T
F	T	T
T	F	F
T	T	T

The AND ($\wedge$)

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

The OR ($\vee$)

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

The IMPLIES ($\implies$)

p	q	$p \implies q$
F	F	T
F	T	T
T	F	F
T	T	T

The IFF ($\iff$)

p	q	$p \iff q$
F	F	T
F	T	F
T	F	F
T	T	T

The NOT ($\neg$)

p	$\neg p$
F	T
T	F

Universality

Every logical sentence can be written using the AND, OR, NOT, IMPLIES, IFF and two more symbols:

There exists, $\exists$

For all, $\forall$

Proof by contradiction

p	q	$p \implies q$
F	F	T
F	T	T
T	F	F
T	T	T

Base b representation

Base b representation

Digits: $0, 1, \dots, b - 1$

Base b representation

Digits: $0, 1, \dots, b - 1$

Represented as $[x]_b$. (Like $[22001]_3$)

Base b representation

Digits: $0, 1, \dots, b - 1$

Represented as $[x]_b$. (Like $[22001]_3$)

Base b representation of a number x is the unique way of writing

$$x = x_0 * b^0 + x_1 * b^1 + \dots + x_k * b^k,$$

where $x_0, x_1, \dots, x_k \in \{0, 1, \dots, (b - 1)\}$

Binary Representation

- When one represent a number in base 2 it is called binary representation.

Binary Representation

- When one represent a number in base 2 it is called binary representation.
- Sometimes called Boolean representation after English mathematician George Boole.

Binary Representation

- When one represent a number in base 2 it is called binary representation.
- Sometimes called Boolean representation after English mathematician George Boole.
- Computer talks in this language.

The world of the computers

The world of the computers

- Every number is stored in binary

The world of the computers

- Every number is stored in binary
- Every number has a certain length (depending of the register size).
For example:
If the register size is 8 then 1 is stored as 00000001.

The world of the computers

- Every number is stored in binary
- Every number has a certain length (depending of the register size).
For example:
If the register size is 8 then 1 is stored as 00000001.
- Cannot store more than a certain number of digits.

Computer Addition

Let the register size in a computer is 8 bits.

Let $x = 11111111$ and $y = 00000001$

Computer Addition

Let the register size in a computer is 8 bits.

Let $x = 11111111$ and $y = 00000001$

What is $x + y$?

Computer Addition

Let the register size in a computer is 8 bits.

Let $x = 11111111$ and $y = 00000001$

What is $x + y$?

Ans: $x + y = 100000000$.

Computer Addition

Let the register size in a computer is 8 bits.

Let $x = 11111111$ and $y = 00000001$

What is $x + y$?

Ans: $x + y = 100000000$.

But the computer sees only the last 8 digits. So it sees 00000000.

Boolean Algebra

Boolean Algebra

- Boolean Algebra has two basic digit: 1 and 0.

Boolean Algebra

- Boolean Algebra has two basic digit: 1 and 0.
- One can think of these as *True* and *False*

Boolean Algebra

- Boolean Algebra has two basic digit: 1 and 0.
- One can think of these as *True* and *False*
- Used to represent data and used in logic.

Representing Data as sets

- Sets

Representing Data as sets

- Sets
 - For example:
 - Set of names of all students

Representing Data as sets

- Sets

For example:

- Set of names of all students
- Set of letters in the english alphabet

Representing Data as sets

- Sets

For example:

- Set of names of all students
- Set of letters in the english alphabet
- Set of digits. $\{0, 1, \dots, 9\}$ or $\{0, 1\}$

Representing Data as sets

- Sets

For example:

- Set of names of all students
- Set of letters in the english alphabet
- Set of digits. $\{0, 1, \dots, 9\}$ or $\{0, 1\}$

- Unordered Sets

Representing Data as sets

- Sets

For example:

- Set of names of all students
- Set of letters in the english alphabet
- Set of digits. $\{0, 1, \dots, 9\}$ or $\{0, 1\}$

- Unordered Sets

- Ordered Sets

(Also called LIST/STRINGS/VECTORS)

Cartesian Product

Cartesian Product

- Let A be a set
- A^n is the set of all ordered subsets (with repetitions) A of size n

Cartesian Product

- Let A be a set
- A^n is the set of all ordered subsets (with repetitions) A of size n
- $\{0, 1\}^n$ the set of all “strings” of 0 and 1 of length n .

A little bit of counting

Q: How many elements are there in the set $\{0, 1\}^n$?

A little bit of counting

Q: How many elements are there in the set $\{0, 1\}^n$?

Ans: 2^n .