

CSE 20

Lecture 15: Midterm Review

Midterm Review

- Representation of integers in base b
- Propositional Logic
- Proof systems:
 - Direct Proof
 - Proof by contradiction
 - Contrapositive
- Sets Theory
- Functions

Representation of integers in base b

- Every integer n can be represented in base b
- In base b the digits are $\{0, 1, \dots, b-1\}$
- Any n can be written in a unique way as

$$x_0b^0 + x_1b^1 + \dots + x_tb^t.$$

- n is written as $[x_0x_1\dots x_t]_b$

Propositional Logic

- Every statement is either TRUE or FALSE
- There are logical connectives $\vee$, $\wedge$, $\neg$, $\implies$ and $\iff$.
- Two logical statements can be equivalent if the two statements answer exactly in the same way on every input.
- To check whether two logical statements are equivalent one can do one of the following:
 - Checking the Truthtable of each statement
 - Reducing one to the other using reductions

Propositional Logic Rules

- Associative Law: $((p \vee q) \vee r) = (p \vee (q \vee r))$ and $((p \wedge q) \wedge r) = (p \wedge (q \wedge r))$
- Commutative law: $(p \vee q) = (q \vee p)$ and $(p \wedge q) = (q \wedge p)$.
- Distributive law: $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ and $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$.
- De Morgan's Law: $\neg(p \vee q) = (\neg p \wedge \neg q)$ and $\neg(p \wedge q) = (\neg p \vee \neg q)$

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If we want to write the expression $p \implies q$ using $\vee$, $\wedge$ and $\neg$ what is the answer:

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- $(p \vee q)$
- $(p \wedge q)$
- $(\neg p \vee q)$
- $(\neg p \wedge q)$
- $(p \vee \neg q)$

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- $(\neg p \vee q)$
- $(\neg p \wedge q)$
- $(p \vee \neg q)$

Predicate Logic

- There are two important symbols: $\forall$ and $\exists$.
- Some statements can be defined using a variable.
- For example: $P_x = "4x^2 + 3 \text{ is divisible by } 5"$
- We can have statements like: $\forall x \in \mathbb{Z}, 4x^2 + 3 \text{ is divisible by } 5$.
- Or $\exists x \in \mathbb{Z}, 4x^2 + 3 \text{ is divisible by } 5$.

Rules of negation

- $\neg(\forall x, P_x) = (\exists x, \neg P_x).$
- $\neg(\exists x, P_x) = (\forall x, \neg P_x).$

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What is the negation of the statement “all human beings can walk on two legs and think logically”

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- No human being can walk on two legs and cannot think logically
- No human being can walk on two legs or cannot think logically
- There exists a human being who can't walk on two legs or cannot think logically
- There exists a human being who can't walk on two legs and cannot think logically
- If someone cannot walk on two legs or cannot think logically then it is not a human being.

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What is the negation of the statement “all human beings can walk on two legs and think logically”

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Proof Techniques

If one has to prove *statement* $A \implies \text{statement}B$ then one of the following steps can be adapted:

- Directly prove $A \implies B$
- Prove that $(\neg B \wedge A) = \text{False}$ (Or gives a contradiction)

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If one has to prove *statement* $A \implies \text{statement}B$ then one of the following steps can be adapted:

- Directly prove $A \implies B$
- Prove that $(\neg B \wedge A) = \text{False}$ (Or gives a contradiction)
- To prove $(\neg B \wedge A) = \text{False}$ one can prove $(\neg B \wedge A) \implies \text{False}$
- $\neg B \implies \neg A$.

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To prove the following statement by contradiction what needs to be proved:

“ $\forall x \in \mathbb{Z}$, x is even then $x^3 - x$ is divisible by 6”

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- $\forall x \in \mathbb{Z}$, if x is even and $x^3 - x$ is not divisible by 6 then we get a contradiction
- If $\exists x \in \mathbb{Z}$ such that x is even and $x^3 - x$ is not divisible by 6 then we get a contradiction
- If there exists $x \in \mathbb{Z}$ such that $x^3 - x$ is not divisible by 6 and x is not even then we have a contradiction.
- $\forall x \in \mathbb{Z}$, x is not even then $x^3 - x$ is not divisible by 6
- If $\exists x \in \mathbb{Z}$ such that x is not even OR $x^3 - x$ is not divisible by 6 then we get a contradiction.

iclicker question

To prove the following statement by contradiction what needs to be proved:

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- $\forall x \in \mathbb{Z}$, if x is even and $x^3 - x$ is not divisible by 6 then we get a contradiction
- If $\exists x \in \mathbb{Z}$ such that x is even and $x^3 - x$ is not divisible by 6 then we get a contradiction
- If there exists $x \in \mathbb{Z}$ such that $x^3 - x$ is not divisible by 6 and x is not even then we have a contradiction.
- $\forall x \in \mathbb{Z}$, x is not even then $x^3 - x$ is not divisible by 6
- If $\exists x \in \mathbb{Z}$ such that x is not even OR $x^3 - x$ is not divisible by 6 then we get a contradiction.