

CSE 20

Lecture 11: Propositional Logic (contd...)

Reminder

Quiz on Monday, February 3rd.

Syllabus:

- Proofs using Direct proof and proof by contradiction techniques
- Proofs of equivalence of logical statements

Proof Techniques

To prove statement B from A :

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- Direct Proof:

$$A \implies B$$

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$(\neg B \wedge A)$ gives a contradiction

Some Number Theory

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If a, b are two integers such that p divides a but does not divide b then p does not divide $(a + b)$.

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To prove by contradiction:

If “ $\exists n \in \mathbb{Z}^+ \forall x > n$ x is not a prime” then a

CONTRADICTION

Infiniteness of Primes

Let “ $\exists n \in \mathbb{Z}^+ \forall x > n \ x \text{ is not a prime}$ ”

Thus there are finitely many primes : let them be

$$p_1, p_2, \dots, p_t$$

With p_t being the largest prime

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Consider the number $(p_1 \times p_2 \times \dots \times p_t) + 1$

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Consider the number $(p_1 \times p_2 \times \dots \times p_t) + 1$

If $(p_1 \times p_2 \times \dots \times p_t) + 1$ is a prime then we get a contradiction as

$$(p_1 \times p_2 \times \dots \times p_t) + 1 > p_t$$

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If $(p_1 \times p_2 \times \cdots \times p_t) + 1$ is not a prime then a prime must divide it.

But all the primes $p_1, p_2, \dots, p_t$ divides $(p_1 \times p_2 \times \cdots \times p_t)$ but not 1.

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So no prime can divide $(p_1 \times p_2 \times \cdots \times p_t) + 1$

Hence $(p_1 \times p_2 \times \cdots \times p_t) + 1$ is a prime.

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- Proof by Contrapositive: $A \implies B$ is same as proving $\neg B \implies \neg A$.

Equivalence of Statement: Question 1

The function

$$\left((p \vee (r \vee q)) \wedge \neg(p \wedge (\neg q \wedge \neg r)) \right)$$

is equal to the function:

- A. $q \vee r$
- B. $\sim p \vee (r \wedge q)$
- C. $(p \vee q) \vee r$
- D. $(p \vee q) \wedge \sim (p \vee r)$
- E. $(p \wedge r) \vee (p \wedge q)$

Rules of Propositional Logic

- ❶ Commutative law:

$$(p \vee q) = (q \vee p) \text{ and } (p \wedge q) = (q \wedge p)$$

- ❷ Associative law:

$$(p \vee (q \vee r)) = ((p \vee q) \vee r) \text{ and} \\ (p \wedge (q \wedge r)) = ((p \wedge q) \wedge r)$$

- ❸ Distributive law:

$$(p \vee (q \wedge r)) = (p \vee q) \wedge (p \vee r) \text{ and} \\ (p \wedge (q \vee r)) = (p \wedge q) \vee (p \wedge r)$$

- ❹ De Morgan's Law:

$$\neg(p \vee q) = (\neg p \wedge \neg q) \text{ and } \neg(p \wedge q) = (\neg p \vee \neg q)$$

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