

CSE105 (Winter 2014): Homework 7

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Due on Wednesday, March 12, 2014.

There is no turnin submission for this assignment. Just bring your homework to class on the due date.

Problem 1

For any string $w = w_1w_2 \dots w_n$, let $w^r = w_n \dots w_2w_1$ be the reverse of w , i.e., w written backward. The reverse of a language $L^r = \{w^r : w \in L\}$ is defined reversing each string individually. Consider the set

$$P_{TM} = \{\langle M \rangle : M \text{ is a TM with alphabet } \{0, 1\} \text{ such that } \mathcal{L}(M)^r = \mathcal{L}(M)\}$$

of all Turing machines whose recognized language is closed under reverse.

- (a) Give a map reduction from A_{TM} to the complement of P_{TM} , and prove that your reduction is correct.
- (b) Give a map reduction from A_{TM} to P_{TM} , and prove that your reduction is correct.
- (c) Based on your answers to part (a) and (b), answer the following questions: Is P_{TM} recognizable? If P_{TM} co-recognizable? Your answer to each question can be “yes”, “no”, or “it does not follow from parts (a) and (b)”. Provide a brief justification to your answers.

Problem 2

Prove or disprove each of the following statements:

- (a) for any language L other than $\emptyset$ and Σ^* , there is a map reduction from L to the complement of L .
- (b) for any language L other than $\emptyset$ and Σ^* , if there is a map reduction from L to the complement of L , then L is either decidable, or neither recognizable nor co-recognizable.
- (c) for any language L other than $\emptyset$ and Σ^* , if L is decidable, then there is a map reduction from L to the complement of L .

Problem 3

Let L be the set of all strings of the form $\langle M_1, M_2 \rangle$ where M_1, M_2 are Turing machines, and $\mathcal{L}(M_1) \subseteq \mathcal{L}(M_2)$.

- (a) Determine if L is recognizable or not, and prove your answer using map reductions.
- (b) Determine if L is co-recognizable or not, and prove your answer using map reductions.