

Name: _____

Student ID: _____

CSE 21A

Practice Midterm #2

February 21, 2013

There are 8 problems. The number of points a problem is worth is shown next to the problem. Show your work. Also, make sure you write legibly so that I have a chance of being able to read your solutions! Additional scratch paper is available at the front of the room. This is a CLOSED BOOK test. However, you may use one 8 1/2 by 11 inch sheet of paper with hand-written notes (on both sides). You can use a calculator if you wish but it shouldn't be necessary since answers can be left in unexpanded form, i.e., using ! and $\binom{x}{y}$. Good luck!

Note: This exam is probably a bit harder than the real Midterm #2 will be!

1. [10 points] If A and B are events in a probability space with $Pr(A) = \frac{1}{2}$, $Pr(B) = \frac{2}{3}$ and $Pr((A \cup B)^c) = \frac{1}{4}$, then what is $Pr(A \cap B)$?

Solution $\frac{5}{12}$.

2. [15 points] 10 identical iPods are distributed randomly to 4 students. What is the probability that each student got at least one iPod?

Solution $\frac{\binom{9}{3}}{\binom{13}{3}}$.

3. [20 points] A hand H of 4 cards is chosen randomly from a standard deck of 52. Let E_1 be the event that H has *at least one* Queen and let E_2 be the event that H has *at least* 2 Queens. What is the conditional probability $Pr(E_2 | E_1)$?

Solution $Pr(E_2 | E_1) = 1 - \frac{4\binom{48}{3}}{\binom{52}{4} - \binom{48}{4}}$.

4. [15 points] An urn contains 2 Red and 5 Blue marbles. A fair coin is flipped. If the flip is Heads then 2 Red marbles are added to the urn. If the flip is Tails then 2 Blue marbles are *removed* from the urn. Now a random marble M is drawn from the urn.

- (a) What is the probability that M is Red?

(b) What is the probability that the flip was Heads given that M is Blue?

Solution (a) $\frac{19}{45}$; (b) $\frac{25}{52}$.

5. [10 points] A fair coin is flipped 3 times. Consider the three events:

- (i) E_1 – The first flip is Heads;
- (ii) E_2 – The first and third flips agree;
- (iii) E_3 – At least two of the flips are Heads.

Which of the 3 pairs of events are *independent*?

Solution The only pair *not* independent is E_1 and E_3 .

6. [20 points] A biased coin C has $Pr(H) = \alpha$ and $Pr(T) = 1 - \alpha$. The coin C is flipped n times. What is the expected number of times that the consecutive sequence HHT occurs? (For example, if the flip sequence were $HHTHHTTHTTTH$ then this number would be 3.)

Solution $(n - 2)\alpha^2(1 - \alpha)$.

7. [20 points] An urn contains 4 Red and 3 White marbles. A random marble M_1 is drawn and a fair coin is flipped. If the flip is Heads then M_1 is put back into the urn. On the other hand, if the flip is Tails, the marble M_1 is *not* put back into the urn. Now another random marble M_2 is drawn from the urn.

- (i) What is $Pr(M_2 = \text{Red})$?
- (ii) What is $Pr(M_1 = \text{Red} \mid M_2 = \text{Red})$?
- (iii) What is $Pr(\text{Flip is Heads} \mid M_2 = \text{White})$?

Solution (i) $\frac{4}{7}$; (ii) $\frac{15}{28}$; (iii) $\frac{1}{2}$.

8. [20 points] The Acme Computer company manufactures hard drives (HD's). It is known that with probability α a randomly selected HD is Good, and with probability $1 - \alpha$, a randomly selected HD is Bad. There is a test T which behaves as follows. If T is applied to a Good HD then with probability β , it says that the HD is Good (so with probability $1 - \beta$, T says that the HD is Bad). On the other hand, if T is applied to a Bad HD then with probability γ , it says that the HD is Bad (and with probability $1 - \gamma$, it says that the HD is Good).

What is the probability that a randomly selected HD is Bad given that the test T says that it is Bad?

Solution $\frac{(1-\alpha)\gamma}{\alpha(1-\beta)+(1-\alpha)\gamma}$.