Name: _____

Student ID:_____

CSE 21A

Midterm #1

January 31, 2013

There are 7 problems. The number of points a problem is worth is shown next to the problem. Show your work. Also, make sure you write legibly so that I have a chance of being able to read your solutions! Additional scratch paper is available at the front of the room. This is a CLOSED BOOK test. However, you may use one 8 1/2 by 11 inch sheets of paper with hand-written notes (on both sides). You can use a calculator if you wish but it shouldn't be necessary since answers can be left in unexpanded form, i.e., using ! and $\binom{x}{y}$.Good luck!

Prob.	Score
1	
2	
3	
4	
5	
6	
7	
Total	

1. [10 points] How many 5-card hands can be formed from an ordinary deck of 52 cards where all 4 suits occur in the hand?

$$4\binom{13}{2}\binom{13}{1}^3$$

4 ways to choose a suit to use twice, $\binom{13}{2}$ ways to choose two cards from that suit, $\binom{13}{1}$ ways to choose one card from the remaining suits

2. [10 points] A shelf contains 15 books. How many ways can 5 books be selected from these 15 with the restriction that no 2 selected books can be adjacent?

(i) $\binom{15}{5}$;

(ii) $\binom{15}{4}$;

(iii) $\binom{10}{5}$;

(iv) $\binom{11}{5}$;

(v) none of the above.

$$\binom{11}{5}$$

5 books means we kill 4 slots leaving 11 slots remaining for 5 books

3. [20 points] How many ways can 10 identical jellybeans be distributed to 2 boys and 3 girls if every girl gets **at least one** jellybean and every boy gets **at most one** jellybean?

$$\binom{9}{2} + 2\binom{8}{2} + \binom{7}{2}$$

One way to give zero jellybeans to boys then $\binom{9}{2}$ ways to give jellybeans to girls (3 bars, $10 - 0 - 4$ stars)
Two ways to give one jellybean to boys then $\binom{8}{2}$ ways to give jellybeans to girls (3 bars, $10 - 1 - 4$ stars)
One way to give two jellybeans to boys then $\binom{7}{2}$ ways to give jellybeans to girls (3 bars, $10 - 2 - 4$ stars)

4. [10 points] How many different arrangements are there using **all** the letters of **VOODOODOLL**?

$$\frac{10!}{2!2!5!}$$

By the bookkeeper rule

5. (a) [5 points] What is the coefficient of x^4y^6 in the expansion of $(2x-3y)^{10}$?

$$2^4 3^6 \binom{10}{4}$$

2^4 from x, 3^6 from y and $\binom{10}{4}$ counting ways to distribute the exponents accordingly.

(b) [5 points] What is the coefficient of $a^2b^3c^4$ in the expansion of $(2a+b+c)^9$?

$$2^2 \frac{9!}{2!3!4!}$$

2^2 from a then the rest from the multinomial coefficient

6. We want to count step-by-step paths between points with integer coordinates. Only two kinds of steps are allowed: a right-step which increments the x coordinate and an up-step which increments the y coordinate.

(a) [10 points] How many paths are there from point $(0, 0)$ to point $(10, 10)$?

$$\binom{20}{10}$$

See homework 2

(b) [10 points] How many paths are there when there is an impassable boulder sitting at point $(4, 5)$?

$$\binom{20}{10} - \binom{9}{4}\binom{11}{5}$$

See homework 2

7. [10 points] The generating function for the sequence $\langle 1, 1, 1, 1, \ldots \rangle$ is $1 + x + x^2 + x^3 \ldots = \frac{1}{1-x}$. What is a formula for the generating function for the sequence $\langle 1, 1, 2, 2, 3, 3, 4, 4, , \ldots \rangle$?

(a) $\frac{1}{(1-x)^2}$

(b) $\frac{x}{(1-x^2)}$

(c) $\frac{1+x}{(1-x^2)^2}$

(d) $\frac{1-x}{(1-x^2)^2}$

(e) None of the above.

(c)

$$2x + 2x^3 + 2x^5 + ... + 2 + 2x^2 + 2x^4 + ...$$
$$= \frac{d}{dx}\frac{1}{1-x^2} + \frac{1}{x}\frac{d}{dx}\frac{1}{1-x^2}$$

$$= \frac{2x}{(1-x^2)^2} + \frac{2}{(1-x^2)^2}$$

Therefore,

$$1 + x + 2x^2 + 2x^3 + ... = \frac{x}{(1-x^2)^2} + \frac{1}{(1-x^2)^2} = \frac{1+x}{(1-x^2)^2}$$

Scratch paper