

CSE21 WI13

**Homework #5**

[Each problem is worth 20 points. This set is challenging!]

**5.1** A hand  $H$  of 3 random cards are dealt from an ordinary deck of 52. Let  $E_1$  denote the event that  $H$  has at least 1 Ace,  $E_2$  denote the event that  $H$  has at least 2 Aces, and let  $E_{AS}$  denote the event that  $H$  includes the Ace of Spades.

- (i) What are  $Pr(E_1)$ ,  $Pr(E_2)$  and  $Pr(E_{AS})$ ?
  - (ii) What is the conditional probability  $Pr(E_2 | E_1)$ ?
  - (iii) What is the conditional probability  $Pr(E_2 | E_{AS})$ ?
- (Are you surprised that the answers to (ii) and (iii) are different?)

**5.2** An urn contains 3 Red and 4 White marbles. A fair coin is flipped. If the flip is Heads then 1 Red and 2 White marbles are added to the urn. On the other hand, if the flip is Tails, then 1 Red and 2 White marbles are *removed* from the urn. Two random marbles are now drawn from the urn without replacement.

- (i) What is the probability that both of the drawn marbles are White?
- (ii) What is the probability that the flip was Heads, given that the two drawn marbles have different colors?

**5.3** Two teams A and B compete in a “best-of-5” competition. This means they play each other until one team has won 3 games. Suppose that for any of the games, the probability that A beats B is  $\alpha$ . What is the probability that A wins the “best-of-5” competition?

**5.4** A fair coin is flipped 3 times. If  $(F_1, F_2, F_3)$  denotes a typical flip sequence, let  $E_1$  denote the event that *at least two* of the  $F_i$ 's are Heads, let  $E_2$  denote the event that *exactly two* of the  $F_i$ 's are Heads, and let  $E_3$  denote the event that all the  $F_i$  are the same. Which of the pairs of these three events are independent?

**5.5** Two random cards are drawn one at a time without replacement from a deck of 52.

- (i) What is the probability that the second card is an Ace?

- (ii) What is the probability that the second card is an Ace, given that the first card drawn was a King?
- (iii) What is the probability that the second card is an Ace, given that the first card drawn was an Ace?

**5.6** A biased coin  $C$  has  $Pr(Heads) = \alpha$  and  $Pr(Tails) = 1 - \alpha$ . The coin is flipped  $n$  times.

What is the expected number of Heads that will occur?

(*Optional*) What is the expected number of times that the sequence  $HT$  will occur? (For example, in the sequence  $HHTTHTHTT$ ,  $HT$  occurs 3 times.)