CSE21 WI13

Homework #4

[Each problem is worth 10 points.]

4.1 A *warped* coin has probability of 0.5 of landing Heads, probability of 0.4 of landing Tails, and probability 0.1 of landing on its Edge. (I said it was warped!). It is flipped 5 times. What is the probability that more Heads occur than Tails?

Solution: We can split this problem into subcases according to the number of Edges that occur:

- No Edges (5, 4 or 3 Heads): $(.5)^5 + \binom{5}{1}(.5)^4(.4) + \binom{5}{2}(.5)^3(.4)^2$.
- One Edge (4 or 3 Heads): $\binom{5}{1}(.5)^4(.1) + \binom{5}{3,1,1}(.5)^3(.4)(.1)$
- Two Edges (3 or 2 Heads): $\binom{5}{2}(.5)^3(.1)^2 + \binom{5}{2,2,1}(.5)^2(.4)(.1)^2$
- Three Edges (2 Heads): $\binom{5}{2}(.5)^2(.1)^3$
- Four Edges (one Head): $\binom{5}{1}(.5)(.1)^4$

For the final answer we sum up all these values, to get $(.5)^5 + \binom{5}{1}(.5)^4(.4) + \binom{5}{2}(.5)^3(.4)^2 + \binom{5}{1}(.5)^4(.1) + \binom{5}{3,1,1}(.5)^3(.4)(.1) + \binom{5}{2}(.5)^3(.1)^2 + \binom{5}{2,2,1}(.5)^2(.4)(.1)^2 + \binom{5}{2}(.5)^2(.1)^3 + \binom{5}{1}(.5)(.1)^4$

4.2 (a) 10 cards are drawn at random one at a time *with replacement* from an ordinary deck of cards.What is the sample space? What is the probability that no Ace appears on any of the draws? What is the probability that at least one King appears in 10 draws? What is the probability that at least 2 Queens appear in the 10 draws?

(b) What are the corresponding probabilities in (a) if the drawing is done *without replacement*?

Solution: (a) The sample space is a sequence of ten cards, and its size is 52^{10};

No Ace has probability $(\frac{48}{52})^{10}$;
By looking at the complement, at least one King has probability $1 - (\frac{48}{52})^{10}$;
There's a $\binom{10}{1}(\frac{4}{52})(\frac{48}{52})^9$ probability of getting exactly one queen, so by compliment at least 2 Queens has probability $1 - (\frac{48}{52})^{10} - \binom{10}{1}(\frac{4}{52})(\frac{48}{52})^9$;
(b) Since we are drawing without replacement, it is easier to think of drawing (unordered) subsets, and the sample space is all hands of ten cards;
Thus, no Ace has probability $\frac{\binom{48}{10}}{\binom{52}{10}}$;
At least one King has probability $1 - \frac{\binom{48}{10}}{\binom{52}{10}}$;
At least 2 Queens has probability $1 - \frac{\binom{48}{10}+4\binom{48}{9}}{\binom{52}{10}}$;

4.3 Let A and B be events with $P(A) = \frac{3}{7}, P(B) = \frac{1}{2}$ and $P((A \cup B)^c) = \frac{3}{8}$. What is $P(A \cap B)$?

Solution: Use a Venn diagram. $Pr(A \cup B) = 1 - Pr((A \cup B)^c = \frac{3}{8}$. Since by inclusion/exclusion, $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$ then $Pr(A \cap B) = Pr(A) + Pr(B) - Pr(A \cup B) = \frac{3}{7} + \frac{1}{2} - \frac{5}{8} = \frac{17}{56}$.

4.4 12 identical jellybeans are distributed randomly to 5 students. What is the probability that each student gets at least one jellybean? What is the probability that each student gets at least two?

Solution: (Bars and stars) – The total number of ways of distributing the jellybeans with no restriction is $\binom{12+4}{4}$. The number of ways of distributing them so that each student gets at least one is $\binom{7+4}{4}$. Therefore the probability we want is $\frac{\binom{11}{4}}{\binom{16}{4}}$. For the case that each student gets at least two, the number of ways to distribute them is $\binom{2+4}{4}$ so the answer is $\frac{\binom{6}{4}}{\binom{16}{4}}$.

4.5 A bin at Blockbuster contains 100 DVDs of which 20 are defective. You randomly select 10 and try them out at home. You discover that there are 2 defective DVDs in the 10 that you selected. The store now allows you to select 2 replacements from the same bin (which now only has 90 DVDs in it, since you already removed 10). What is the probability that *none* of 10 DVDs you finally end up with are defective? What is the answer to this question if you find k defectives in your initial choice?

Solution: We will ignore the 2 defective DVDs in our initial selection, because we will get to replace them. Remaining in the bin after our first random pick of 10 DVDs are 90 DVDs, 18 of which are defective, but 72 good. So the probability that *none* of the DVDs out of the 2 additional ones we select are defective is $\frac{\binom{72}{2}}{\binom{90}{2}}$. In general, the answer would be $\frac{\binom{70+k}{k}}{\binom{90}{k}}$

4.6 An urn contains 2 Red marbles, 3 White marbles and 4 Blue marbles. You reach in and draw out 3 marbles at random (*without replacement*). What is the probability that you will get one marble of each color? What is the answer if you draw out the marbles one at a time *with replacement*?

Solution: (a) There $\binom{9}{3}$ ways we can draw 3 marbles out of an urn of 9 marbles, this corresponds to the size of the sample space. If we draw one of each color, there are $\binom{2}{1}$ possibilities for the Red one, $\binom{3}{1}$ possibilities for the White one and $\binom{4}{1}$ for the Blue one, then we apply the product rule and the final answer is $\frac{\binom{2}{1}\binom{3}{1}\binom{4}{1}}{\binom{9}{3}} = \frac{2}{7}$;
(b) The sample space is now all sequences of 3 marbles, and RWB and RBW will correspond to separate outcomes. Therefore, the answer now is $3! \cdot (\frac{2}{9})(\frac{3}{9})(\frac{4}{9}) = \frac{16}{81}$.