

CSE21 WI13

Homework #2

[Each problem is worth 10 points.]

2.1 Let S be a set of size $n > 0$. Show that the number of subsets of S with an *even* number of elements is equal to the number of subsets of S with an *odd* number of elements.

For example, for $S = \{a, b, c\}$, the even subsets are $\emptyset, \{a, b\}, \{a, c\}, \{b, c\}$ and the odd subsets are $\{a\}, \{b\}, \{c\}, \{a, b, c\}$.

(Hint: Think about the binomial theorem.)

2.2 How many 5-card hands from an ordinary deck of 52 have all cards with the same color (i.e., all red = \diamond, \heartsuit or all black = \clubsuit, \spadesuit)?

2.3 (a) How many paths are there from point $(0,0)$ to $(50,50)$ if every step increments one coordinate by one unit and leaves the other unchanged.

(b) How many are there when there are impassable boulders sitting at points $(10,11)$ and $(21,20)$?

(You do not have to calculate the number explicitly but rather you can leave the answer expressed in terms of binomial coefficients).

2.4 (a) How many ways are there of distributing 5 oranges and 7 bananas to 4 (distinct) students?

(b) What is the answer if each student must get at least one banana.?

(c) (Extra credit) What is the answer if all you require is that each student gets at least one piece of fruit (orange or banana)?

2.5 The working days in the next year can be numbered $1, 2, \dots, 300$. I'd like to avoid as many as possible.

- On even-numbered days, I'll say I'm sick;
- On day that are a multiple of 3, I'll say I was stuck in traffic;
- On days that are a multiple of 5, I'll refuse to come out from under the blankets.

In total, how many work days will I *avoid* in the coming year?

2.6 Show that in any set of 100 integers there is always some pair whose difference is divisible by 99.