CSE21 WI13

Homework #1

[Each problem is worth 10 points.]

1.1 Suppose $n > 1$. An n-digit number is a string of n digits from $\{0, 1, 2, \ldots, 9\}$ where the first digit is not 0.

(a) How many n-digit numbers (as a function of n?

There are 9 choices for the first digit and 10 for every remaining digit so there are $9 \cdot 10^{n-1}$ n-digit numbers by the product rule.

(c) How many of these *don't* contain the digit 4?

We use part (c) to solve part (b). Without using the digit 4, we have 8 choices for the first digit and 9 for every remaining digit for a total of $8 \cdot 9^{n-1}$.

(b) How many of these contain the digit 3?

Similar to above, there are $8 \cdot 9^{n-1}$ n-digit numbers that don't contain 3. With $9 \cdot 10^{n-1}$ numbers total, there are $9 \cdot 10^{n-1} - 8 \cdot 9^{n-1}$ n-digit numbers that do not contain 3.

1.2 In a certain computer system, a valid password consists of a string of 5, 6, or 7 symbols. The first symbol must be an uppercase letter. The remaining symbols can only be numbers or lowercase letters.
How many valid passwords are there?

There are 26 choices for the first character and 36 for all remaining characters for a total of $26 \cdot 36^4$ 5 digit passwords, $26 \cdot 36^5$ 6 digit passwords, and $26 \cdot 36^6$ 7 digit passwords, for a total of $26 \cdot 36^4 + 26 \cdot 36^5 + 26 \cdot 36^6$ passwords by the sum rule.

1.3 A license plate consists of either:

- 3 letters followed by 4 numbers (standard plate)
- 5 letters (vanity plate)
- 2 characters, letters or numbers (big shot plate)

How many possible license plates are there?

standard plate: 26 choices for the first three characters, 10 for the last four characters
vanity plate: 26 choices for each character
big shot plate: 36 choices for each character
for a total of $26^3 \cdot 10^4 + 26^5 + 36^2$ liscense plates.

1.4 We would like to count how many ways 3 boys and 3 girls can sit in a row.

How many ways can this be done if:

(a) there are no restrictions?

Assuming each person is a unique individual, and each child identifies with a single gender:
There are 6! ways to arrange 6 unique people. Therefore there are 6! ways to arrange 3 boys and 3 girls.

(b) all the girls sit together?

Since all the girls must sit together, we treat the girls as a single unit. Then we have 4 people to arrange with 3! positions for 3 girls for a total of 4!3! ways to arrange them.

(c) every boy sits next to at least one other girl?

If three boys sit next to each other, no combinations work. If two boys sit next to each other, it fails if and only if the pair of boys sitting next to each other are on an edge (ie. BBGGBG, BGGGBB). If no two boys sit next to each other, all combinations work. There are 4!3! combinations with three boys together (see part (b)). If we place two boys on the edge, we have two choices, left or right to place them. We then choose the position of the third boy from threeremaining positions (he can't be next to the two other boys) for a total of $2 \cdot 3 \cdot 3!3!$ positions (3!3! to account for varying positions of unique boys and girls). Since there are 6! total positionings, there are $6! - 4!3! - 2 \cdot 3 \cdot 3!3! = 360$ positionings where no two boys sit next to each other.

1.5 Lisa is thinking of a number between 1 and 1000. What is the least number of yes/no questions you could ask her and be guaranteed of learning her numbers? (Lisa always answers your questions truthfully).

Each question has two possible outcomes so we require $\lceil log_2(1000) \rceil = 10$ questions to guess the number. This problem is similar to a binary search. A set of 10 questions could be:
In the binary representation of your number, is the most significant bit 1?
In the binary representation of your number, is the second most significant bit 1?
...
In the binary representation of your number, is the least significant bit 1?

1.6 How many ways are there of placing a King, a Queen, a Knight and a Bishop on a (standard) chessboard so that no two pieces are in the same row or column?

We first place the king on an 8x8 board. Since the queen cannot be in the same row or column as the piece already on the board, one row and one column is eliminated effectively limiting the queen to a 7x7 board. The knight then is placed on a 6x6 board and the bishop on a 5x5 board for a total of $8^2 7^2 6^2 5^2$ possible positions.