CSE 21A

Practice Problems

March 8, 2013

The following problems are good practice for the Final Exam. It is very likely the almost all of the problems on the Final will be closely related to some of these problems. I'll post annotated solutions within the next week.

1. If A and B are events in a probability space with $Pr(A) = \frac{1}{3}$, $Pr(B) = \frac{1}{4}$ and $Pr((A \cap B)^c) = \frac{11}{12}$, then what is $Pr((A \cup B)^c)$?

2. 8 Red and 9 Blue jellybeans are distributed randomly to 4 students. What is the probability that each student got *at least one* jellybean of each color?

3. How many 5-card hands can be formed from an ordinary deck of 52 cards if *exactly two* suits are present in the hand?

4. A shelf contains 24 books. How many ways can 6 books be selected from these 24 with the restriction that no two selected books can be adjacent?

5. A hand H of 5 cards is chosen randomly from a standard deck of 52. Let E_1 be the event that H has *at least one* King and let E_2 be the event that H has *at least* 2 Kings. What is the conditional probability $Pr(E_2 \mid E_1)$?

6. An urn contains r Red and b Blue marbles. A fair coin is flipped. If the flip is Heads then h Red marbles are added to the urn. if the flip is Tails then t Blue marbles are add to the urn. Now a random marble M is drawn from the urn.

(a) What is the probability that M is Red?

(b) What is the probability that the flip was Heads given that M is Blue?

7. A fair coin is flipped 3 times resulting in the flip sequence $F_1F_2F_3$.. Consider the three events:
(i) $E_1 - \{F_1$ is Heads$\}$;
(ii) $E_2 - \{F_2$ and F_3 agree$\}$;
(iii) $E_3 - \{F_1$ and F_3 disagree.$\}$

Which of the 3 pairs of events are *independent?*

8. A biased coin C has $Pr(H) = \alpha$ and $Pr(T) = 1 - \alpha$. The coin C is flipped n times. What is the expected number of times that the consecutive sequence HXH occurs where X can be either H or T? (For example, if the flip sequence were $HHTHHHTHTHHT$ then this number would be 4.)

9. An urn contains r Red and w White marbles. A random marble M_1 is drawn and a fair coin is flipped. If the flip is Heads then M_1 is put back into the urn. On the other hand, if the flip is Tails, the marble M_1 is *not* put back into the urn. Now another random marble M_2 is drawn from the urn.

(i) What is $Pr(M_2 = \text{Red})$?
(ii) What is $Pr(M_1 = \text{Red} \mid M_2 = \text{Red})$?
(iii) What is $Pr(\text{Flip is Heads} \mid M_2 = \text{White})$?

10. How many different way are there of arranging **all** the letters of the string **CALCULUSBOOK**?

11. (a) What is the coefficient of x^5 in the expansion of $(3x-1)^{11}$?

12. We want to count step-by-step paths between points with integer coordinates. Only two kinds of steps are allowed: a right-step which increments the x coordinate by 1 and an up-step which increments the y coordinate by 1.

(i) How many paths are there from point $(0,0)$ to point $(10,10)$?

(ii) How many paths are there if there is an impassable boulder sitting at point $(5,6)$?

(iii) How many paths are there if there are impassable boulders sitting at points $(2,3)$ and $(6,8)$?

13. The Acme Tire company manufactures tires. It is known that with probability α a randomly selected tire is good, and with probability $1-\alpha$, a randomly selected tire is bad. There is a test T which behaves as follows. If T is applied to a good tire then with probability β, it says that the tire is good (so with probability $1-\beta$, T says that the tire is bad). On the other hand, if T is applied to a bad tire then with probability γ, it says that the tire is bad (and with probability $1-\gamma$, it says that the tire is good).
What is the probability that a randomly selected tire is good given that the test T says that it is bad?

14. The generating function for the sequence $\langle 1,1,1,1,\ldots\rangle$ is $1+x+x^2+x^3\ldots = \frac{1}{1-x}$.
What is the generating function for the sequence $\langle 1,1,3,3,5,5,7,7,9,9,\ldots\rangle$?

15. An urn contains 1 Red ball and n White balls. You repeatedly draw balls *without replacement* until you get the Red ball. What is the expected number of draws until this happens?
What is the answer if instead you draw *with replacement*?

16. A sequence is defined by: $a(1) = 1$ and $a(n+1) = 3a(n) - 1$ for $n \geq 1$. What is $a(100)$?

17. How many sequences of length n made up of 1, 2 and 3 do not have *two consecutive repeated* symbols?
(For example, 012120210212 would be allowed but 012112021021 would not.)

18. What is the general solution to the recurrence: $t(n+2) = 2t(n+1) + 2t(n)$, $n \geq 0$, with $t(0) = 0, t(1) = 1$?

19. What is the general solution to the recurrence: $x(n+2) = 3x(n+1) - 2x(n) + n$, $n \geq 0$, with $x(0) = 0, x(1) = 1$? (Hint: Try a *quadratic* polynomial for the specific solution to the recurrence.)

20. What is the general solution to the recurrence: $x(n+2) = 6x(n+1) - 9x(n)$, $n \geq 0$, with $x(0) = 0, x(1) = 1$?

21. An urn contains 6 Red balls and 1 Blue ball. A fair die having faces $\{1, 2, 3, 4, 5, 6\}$ is rolled. It the top face on the die shows m, then m random balls are removed from the urn. What is the expected number of Red balls iremoved by this process?

22. A bin contains contains 10 Orange and 12 Green marbles. A random set S of 6 marbles are removed from the bin *without replacement*. What is the probability that S contains *at least* 2 Orange marbles, given that S contains an Orange marble? What is the probability that that S contains *at least* 2 Orange marbles, given that S contains a Green marble?

23. In how many ways can 10 identical cookies be distributed to 2 boys and 3 girls if no boy gets more than 1 cookie and every girl gets at least 1 cookie?

24. A valid password P consists of 5 characters taken from the sets of 26 letters $\{A, B, C, \ldots, Z\}$ and 10 numbers $\{0, 1, 2, \ldots, 9\}$. However, P must have at least one number and at least one number, and furthermore, P cannot have both of the symbols O and 0 in it. How many valid passwords are there?

25. What is the length of the Minimum Spanning Tree for the following weighted graph?

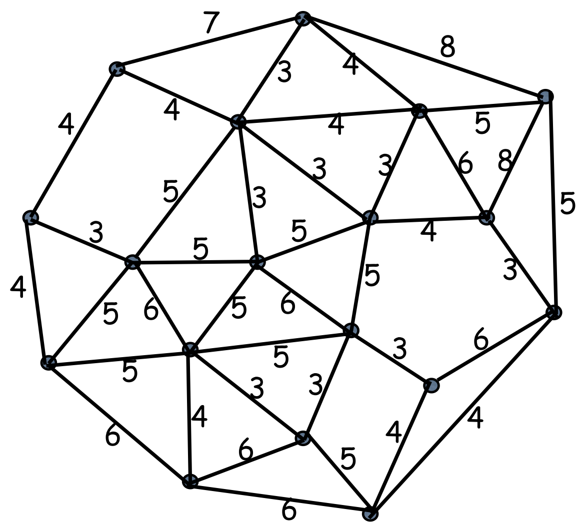

Figure 1: A weighted graph.