

CSE 203 Homework 1
Approximation algorithms, basic techniques
Due Tuesday, Feb. 5

General instructions: Each problem is worth 10 points. For each of the algorithm problems, design a polynomial-time approximation algorithm with a performance ratio as indicated. Give a proof of the approximation guarantee and time analysis for each algorithm. You may use any well-known algorithm or data structure, or algorithm from the texts or from class, as a sub-routine, without needing to provide details. 3. Don't do literature searches, just use the textbooks.

Average Finish Time The *Minimum Average Finish Time Schedule on Two Machines* is as follows: You are given a list of n jobs of positive real durations $d_1, \dots, d_n$. You must assign each job to one of two identical machines, and order the jobs on each machine. The *finish time* of a job i is the sum of all the durations of all jobs assigned before it on the same machine, plus its own duration. The objective is to minimize the *sum* of finish times of all jobs. Give a fully polynomial-time approximation scheme (FPTAS) for this problem.

Heterogenous Load Balancing Consider a heterogenous version of the load balancing problem where each of m processors has a speed, s_i (in cycles/unit time). Each job has a requirement T_i in cycles, and needs to be performed by exactly one processor. We want to minimize the time when all jobs are finished, i.e., $\max_{1 \leq i \leq m} (\sum_{\text{jobs } j \text{ assigned to processor } i} T_j) / s_i$.

Give an efficient approximation algorithm for this problem. (Hint: the following has ratio 2: Sort the jobs from largest to smallest. Assign each job to the machine that would finish it first, i.e., if machine i would finish its current jobs at F_i , minimize $F_i + T_j / s_i$.)

Bounding Sphere Say that $p_1, \dots, p_n$ are points in R^d , the d -dimensional Euclidean space, i.e., each p_i is a vector of d real numbers. A bounding sphere $S(q, R)$ is a sphere centered at point q (which is not necessarily one of the input points) of radius R that contains all of the p_i , i.e., the distance between q and every p_i is at most R . The Minimum Radius Bounding Sphere problem is, given such a set of points, find the bounding sphere with smallest radius R . Give a polynomial time approximation algorithm for this problem with ratio strictly less than 2. (Note: we will return to this problem after we study convex optimization.)

Steiner trees for [1,2]-metrics A metric space on a set of n elements $x_1 \dots x_n$ is a distance function $d(x_i, x_j)$ satisfying $d(x, x) = 0$ and the triangle inequality $d(x_i, x_j) \leq d(x_i, x_k) + d(x_k, x_j)$ for every three points x_i, x_j, x_k . The Steiner problem is : given a metric space and a subset R of required

elements, find a tree that contains all the required elements (and possibly some others) that minimizes the total distances along all edges. A $[1,2]$ -metric space is one where $1 \leq d(x_i, x_j) \leq 2$ for every two distinct points x_i, x_j . Note that this already implies the triangle inequality. Give a polynomial-time approximation algorithm to the Steiner problem for $[1,2]$ -metric spaces that has approximation ratio strictly less than 2.