

Motion

Introduction to Computer Vision
CSE 152
Lecture 20

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Intro Computer Vision

Announcements

- Final exam Q&A session: Monday, 3:30-4:30, CSE Room 4217
- Final exam: Tuesday 3/19 8:00AM

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- Continuous motion vs. Wide baseline
- Number of features + measurements
- Two view Structure from Motion (SFM)
- Object + Motion Detection: frame differences, background subtraction
- The Motion Field
- Optical flow

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Discrete Motion: Some Counting

Consider M images of N points, how many unknowns?

1. Camera locations: Affix coordinate system to location of first camera location: $(M-1)*6$ Unknowns
2. 3-D Structure: $3*N$ Unknowns
3. Can only recover structure and motion up to scale. Why?

Total number of unknowns: $(M-1)*6+3*N-1$

Total number of measurements: $2*M*N$

Solution is possible when $(M-1)*6+3*N-1 \leq 2*M*N$

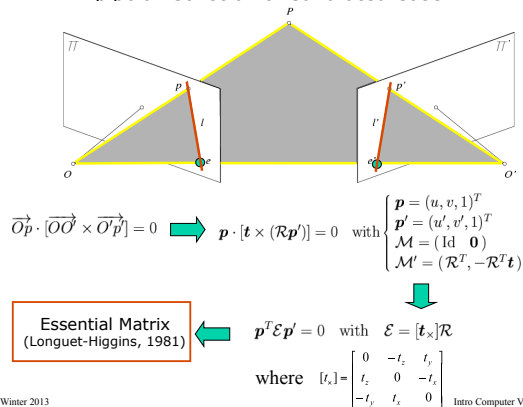
$M=2 \rightarrow N \geq 5$

$M=3 \rightarrow N \geq 4$

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Epipolar Constraint: Calibrated Case



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The Eight-Point Algorithm (Longuet-Higgins, 1981)

Let $\mathbf{F}$ denote the Essential Matrix here

$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Rightarrow (uu', uv', u, vu', vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

Set F_{33} to 1

$$\begin{pmatrix} u_1 u'_1 & u_1 v'_1 & u_1 & v_1 u'_1 & v_1 v'_1 & v_1 & u'_1 & v'_1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 u'_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 \\ u_3 u'_3 & u_3 v'_3 & u_3 & v_3 u'_3 & v_3 v'_3 & v_3 & u'_3 & v'_3 \\ u_4 u'_4 & u_4 v'_4 & u_4 & v_4 u'_4 & v_4 v'_4 & v_4 & u'_4 & v'_4 \\ u_5 u'_5 & u_5 v'_5 & u_5 & v_5 u'_5 & v_5 v'_5 & v_5 & u'_5 & v'_5 \\ u_6 u'_6 & u_6 v'_6 & u_6 & v_6 u'_6 & v_6 v'_6 & v_6 & u'_6 & v'_6 \\ u_7 u'_7 & u_7 v'_7 & u_7 & v_7 u'_7 & v_7 v'_7 & v_7 & u'_7 & v'_7 \\ u_8 u'_8 & u_8 v'_8 & u_8 & v_8 u'_8 & v_8 v'_8 & v_8 & u'_8 & v'_8 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \text{Solve For F} \Rightarrow \text{Solve For R and t}$$

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Sketch of Two View SFM Algorithm

Input: Two images

1. Detect feature points
2. Find 8 matching feature points (easier said than done, but usually done with RANSAC Algorithm)
3. Compute the Essential Matrix E using Normalized 8-point Algorithm
4. Compute R and T (recall that $E=RS$ where S is skew symmetric matrix)
5. Perform stereo matching using recovered epipolar geometry expressed via E .
6. Reconstruct 3-D geometry of corresponding points.

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Continuous Motion

- Consider a video camera moving continuously along a trajectory (rotating & translating).
- How do points in the image move
- What does that tell us about the 3-D motion & scene structure?



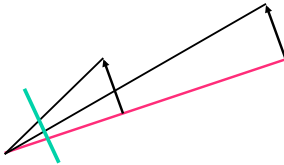
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Motion

“When objects move at equal speed, those more remote seem to move more slowly.”

- Euclid, 300 BC



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Simplest Idea for video processing Image Differences: Brightness change at a location

- Given image $I(u,v,t)$ and $I(u,v, t+\delta t)$, compute $I(u,v, t+\delta t) - I(u,v,t)$.

- This is partial derivative: $\frac{\partial I}{\partial t}$

- At object boundaries, $\left| \frac{\partial I}{\partial t} \right|$ is large and is a cue for segmentation
- Doesn't tell which way stuff is moving

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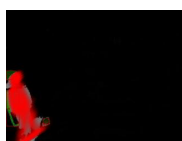
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Background Subtraction

- Gather image $I(x,y,t_0)$ of background without objects of interest (perhaps computed over average over many images).
- At time t , pixels where $|I(x,y,t) - I(x,y,t_0)| > \tau$ are labeled as coming from foreground objects



Raw Image



Foreground region

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Is motion estimation inherent in humans?

Demo

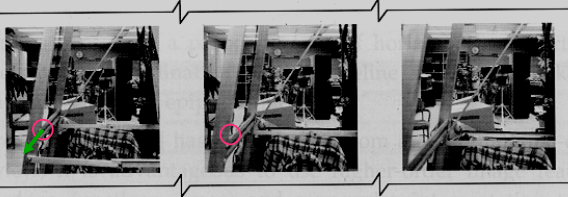


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The Motion Field

Where in the image did a point move?

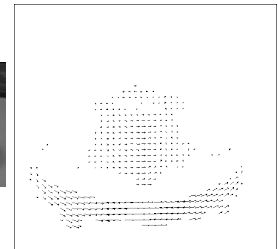
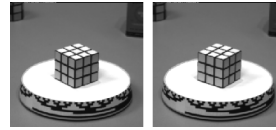


Down and left

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Motion Field



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What causes a motion field?

1. Camera moves (translates, rotates)
2. Object's in scene move rigidly
3. Objects articulate (pliers, humans, quadrupeds)
4. Objects bend and deform (fish)
5. Blowing smoke, clouds

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Optical Flow Field: Looming

- <https://www.youtube.com/watch?v=xTCNBt1OsIU>

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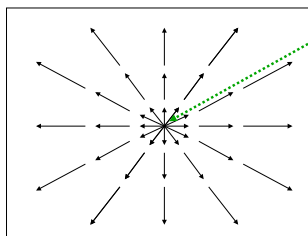
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Motion Field Yields 3-D Motion Information

The "instantaneous" velocity of points in an image

LOOMING

The Focus of Expansion (FOE)



Intersection of velocity vector with image plane

With just this information it is possible to calculate:

1. Direction of motion
2. Time to collision

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on

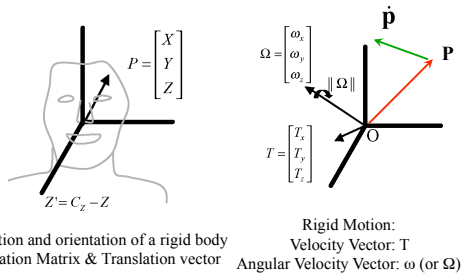
Rigid Motion and the Motion Field

How do we relate 3-D motion of a camera to the 2-D motion of points in the image

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Rigid Motion: General Case



$$\dot{\mathbf{p}} = \mathbf{T} + \omega \times \mathbf{p}$$

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General Motion

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \frac{f}{z} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} - \frac{f\dot{z}}{z^2} \begin{bmatrix} x \\ y \end{bmatrix}$$

Let (x, y, z) be functions of time $(x(t), y(t), z(t))$:

$$\begin{aligned} \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} &= \frac{f}{z} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} - \frac{f\dot{z}}{z^2} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= \frac{f}{z} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} - \frac{\dot{z}}{z} \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

Substitute $\dot{\mathbf{p}} = \mathbf{T} + \omega \times \mathbf{p}$ where $\mathbf{p} = (x, y, z)^T$

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Motion Field Equation

$$\begin{aligned} \dot{u} &= \frac{T_z u - T_x f}{Z} - \omega_y f + \omega_z v + \frac{\omega_x u v}{f} - \frac{\omega_y u^2}{f} \\ \dot{v} &= \frac{T_z v - T_y f}{Z} + \omega_x f - \omega_z u - \frac{\omega_y u v}{f} - \frac{\omega_x v^2}{f} \end{aligned}$$

- $\mathbf{T}$: Components of 3-D linear motion
- ω : Angular velocity vector
- (u, v) : Image point coordinates
- Z : depth
- f : focal length

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Pure Translation

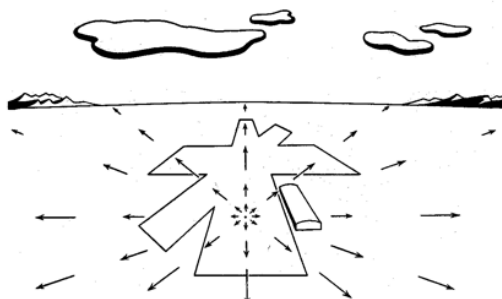
$$\begin{aligned} \dot{u} &= \frac{T_z u - T_x f}{Z} - \omega_y f + \omega_z v + \frac{\omega_x u v}{f} - \frac{\omega_y u^2}{f} \\ \dot{v} &= \frac{T_z v - T_y f}{Z} + \omega_x f - \omega_z u - \frac{\omega_y u v}{f} - \frac{\omega_x v^2}{f} \end{aligned}$$

$$\omega = 0$$

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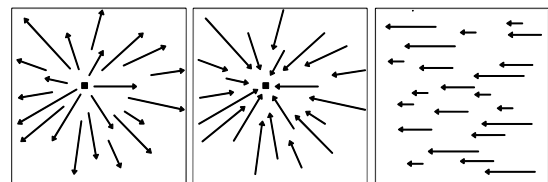
Forward Translation & Focus of Expansion [Gibson, 1950]



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Pure Translation



Radial about FOE

Parallel (FOE point at infinity)
 $T_z = 0$
Motion parallel to image plane

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Pure Rotation: $\mathbf{T}=0$

$$\dot{u} = \frac{T_z u - T_x f}{Z} - \omega_y f + \omega_z v + \frac{\omega_x uv}{f} - \frac{\omega_y u^2}{f}$$

$$\dot{v} = \frac{T_z v - T_y f}{Z} + \omega_x f - \omega_z u - \frac{\omega_y uv}{f} - \frac{\omega_x v^2}{f}$$

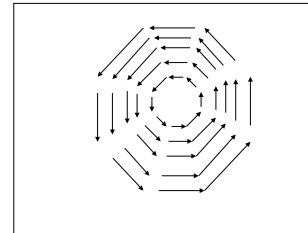
- Independent of $T_x T_y T_z$
- Independent of Z
- Only function of (u,v) , f and ω

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Rotational MOTION FIELD

The "instantaneous" velocity of points in an image



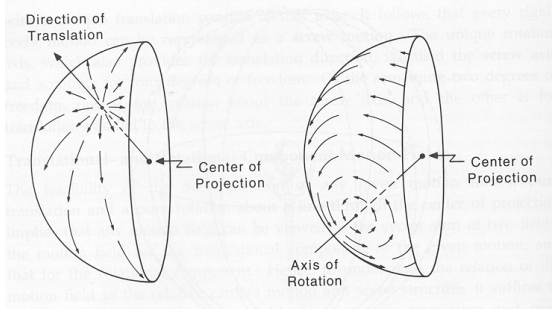
PURE ROTATION

$$\omega = (0,0,1)^T$$

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Pure Rotation: Motion Field on Sphere



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Motion Field Equation: Estimate Depth

$$\dot{u} = \frac{T_z u - T_x f}{Z} - \omega_y f + \omega_z v + \frac{\omega_x uv}{f} - \frac{\omega_y u^2}{f}$$

$$\dot{v} = \frac{T_z v - T_y f}{Z} + \omega_x f - \omega_z u - \frac{\omega_y uv}{f} - \frac{\omega_x v^2}{f}$$

If $\mathbf{T}$, ω , and f are known or measured, then for each image point (u,v) , one can solve for the depth Z given measured motion $(du/dt, dv/dt)$ at (u,v) .

$$Z = \frac{T_z u - T_x f}{\dot{u} + \omega_y f - \omega_z v - \frac{\omega_x uv}{f} + \frac{\omega_y u^2}{f}}$$

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Optical Flow

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Optical Flow: Where do pixels move to?



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Estimating the motion field from images

1. Feature-based (Sect. 8.4.2 of Trucco & Verri)
 1. Detect Features (corners) in an image
 2. Search for the same features nearby (Feature tracking).
2. Differential techniques (Sect. 8.4.1)

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Video: $I(x,y,t)$

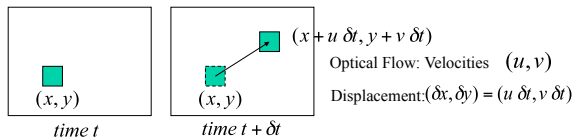
$I(x,y,t)$: brightness at image point (x,y) at time t



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Optical Flow Constraint Equation



- Assume brightness of patch remains same in both images:

$$I(x + u \delta t, y + v \delta t, t + \delta t) = I(x, y, t)$$

- Assume small motion: (Taylor expansion of LHS upto first order)

$$I(x, y, t) + \delta x \frac{\partial I}{\partial x} + \delta y \frac{\partial I}{\partial y} + \delta t \frac{\partial I}{\partial t} = I(x, y, t)$$

- Subtract $I(x, y, t)$ from both sides, divide by δt , and consider δt to be small

$$\frac{dI}{dt} = \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

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Solving for flow

Optical flow constraint equation :

$$\frac{dI}{dt} = \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

- We can measure $\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}, \frac{\partial I}{\partial t}$
- We want to solve for $\frac{dx}{dt}, \frac{dy}{dt}$
- One equation, two unknowns

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Optical Flow Constraint Equation

$$\delta x \frac{\partial I}{\partial x} + \delta y \frac{\partial I}{\partial y} + \delta t \frac{\partial I}{\partial t} = 0$$

Divide by δt and take the limit $\delta t \rightarrow 0$

$$\frac{dx}{dt} \frac{\partial I}{\partial x} + \frac{dy}{dt} \frac{\partial I}{\partial y} + \frac{\partial I}{\partial t} = 0$$

Constraint Equation

$$I_x u + I_y v + I_t = 0$$

Unknown Flow vector

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}$$

Measurements

$$I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y}, \quad I_t = \frac{\partial I}{\partial t}$$

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Aperture Problem and Normal Flow

Measurements

$$I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y}, \quad I_t = \frac{\partial I}{\partial t}$$

$$I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y}, \quad I_t = \frac{\partial I}{\partial t}$$

$$I_x = \frac{\partial I}{\partial x}, \quad I_y = \frac{\partial I}{\partial y}, \quad I_t = \frac{\partial I}{\partial t}$$

The gradient constraint:

$$I_x u + I_y v + I_t = 0$$

$$\nabla I \cdot \vec{U} = 0$$

Defines a line in the (u,v) space

Flow vector

$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}$$

Normal Flow:

$$u_{\perp} = - \frac{I_t \nabla I}{|\nabla I|^2}$$

The component of the optical flow in the direction of the image gradient.

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Normal Flow

Illusion Works Barber Pole Illusion



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Lucas-Kanade: Integrate over a Patch

Assume a single velocity for all pixels within an image patch

$$E(u, v) = \sum_{x, y \in \Omega} (I_x(x, y)u + I_y(x, y)v + I_t)^2$$

$$\frac{dE(u, v)}{du} = \sum 2I_x(I_x u + I_y v + I_t) = 0$$

$$\frac{dE(u, v)}{dv} = \sum 2I_y(I_x u + I_y v + I_t) = 0$$



Solve with:

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = - \begin{pmatrix} \sum I_x I_t \\ \sum I_y I_t \end{pmatrix}$$

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Final Exam

- Closed book
- No Calculator
- One cheat sheet
 - Single piece of paper, handwritten, no photocopying, no physical cut & paste.
 - You can start with your sheet from the midterm.
- What to study
 - Basically material presented in class, and supporting material from text
 - If it was in text, but NEVER mentioned in class, it is very unlikely to be on the exam
- Question style:
 - Short answer
 - Some longer problems to be worked out.

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