

# Stereo Vision I

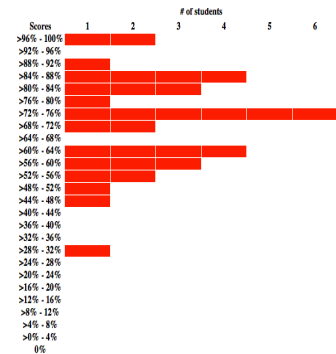
Introduction to Computer Vision  
CSE 152  
Lecture 13

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## Midterm Stats

| Statistics              |           |
|-------------------------|-----------|
| Percent of Course:      | 20.00%    |
| Total Number of Scores: | 31        |
| Mean:                   | 70.50     |
| Median:                 | 72.09     |
| High Score:             | 96.51 (2) |
| Low Score:              | 29.07 (1) |
| Standard Deviation:     | 15.67     |



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## Stereo Vision Outline

- Offline: Calibrate cameras & determine “epipolar geometry”
- Online
  - Acquire stereo images
  - Rectify images to convenient epipolar geometry
  - Establish correspondence
  - Estimate depth

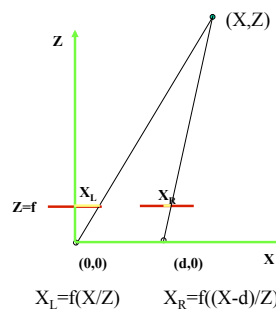
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## BINOCULAR STEREO SYSTEM

Estimating Depth

2D world with 1-D image plane



Two measurements:  $X_L, X_R$   
Two unknowns:  $X, Z$

Constants:

Baseline:  $d$

Focal length:  $f$

$$X = \frac{df X_L}{(X_L - X_R)}$$

$$Z = \frac{df}{(X_L - X_R)}$$

Disparity:  $(X_L - X_R)$

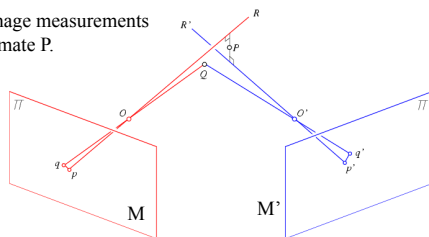
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(Adapted from G> Hager)

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## Reconstruction: General 3-D case

Given two image measurements  $p$  and  $p'$ , estimate  $P$ .



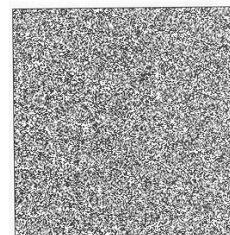
- Linear Method: find  $P$  such that
 
$$\begin{cases} p \times MP = 0 \\ p' \times M'P = 0 \end{cases} \iff \begin{pmatrix} p \times \\ p' \times \end{pmatrix} \begin{bmatrix} M \\ M' \end{bmatrix} P = 0$$
 Where  $M$  and  $M'$  are the two camera matrices

- Non-Linear Method: find  $Q$  minimizing  $d^2(p, q) + d^2(p', q')$  where  $q=MQ$  and  $q'=M'Q$

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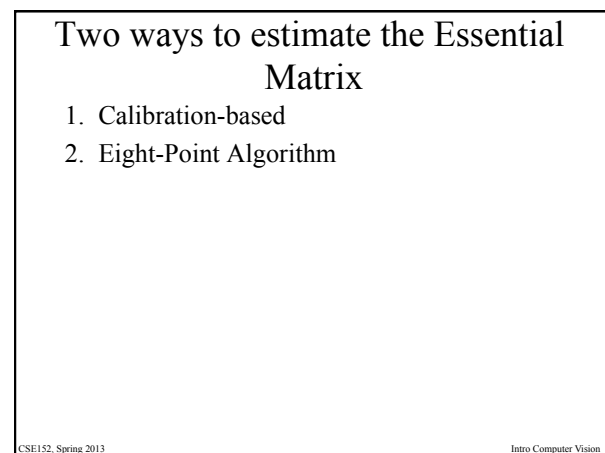
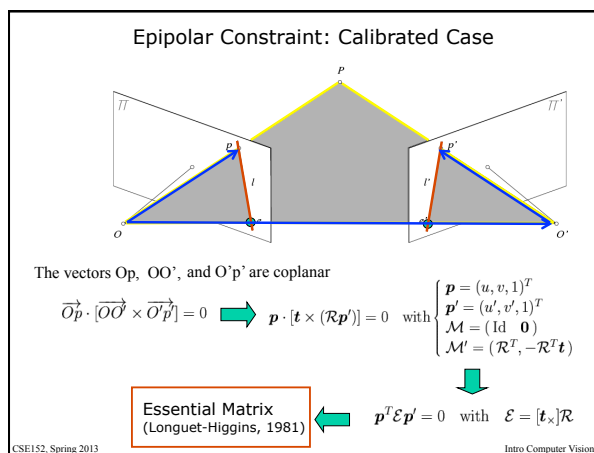
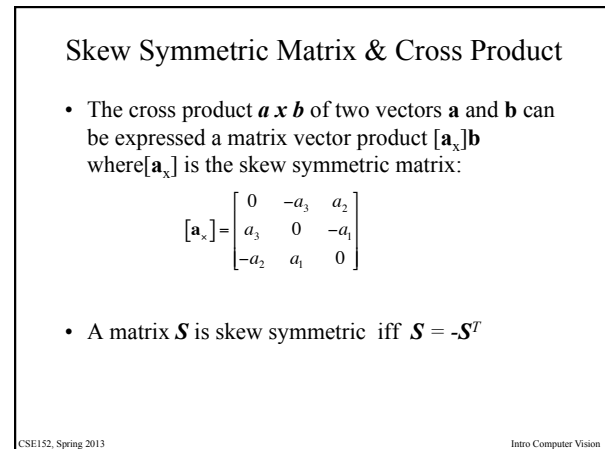
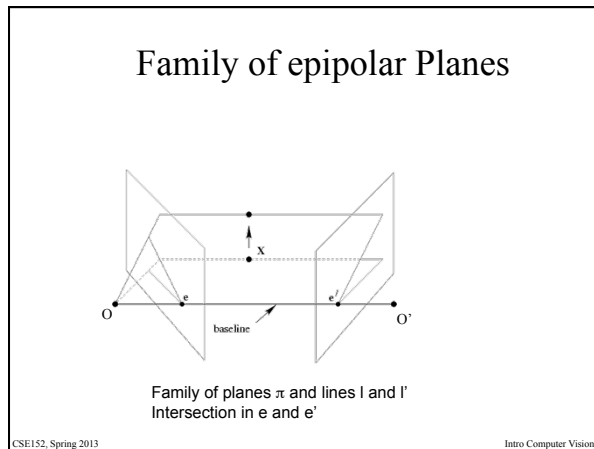
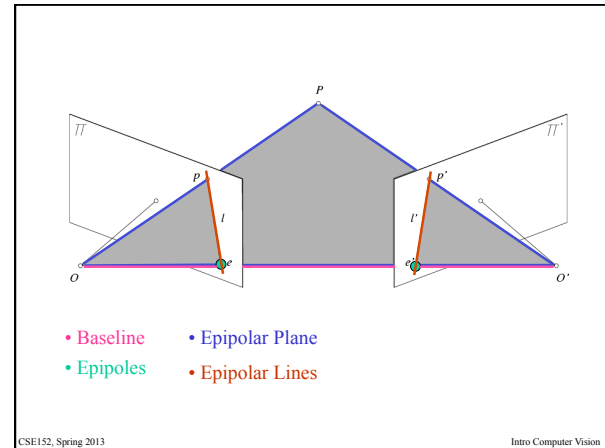
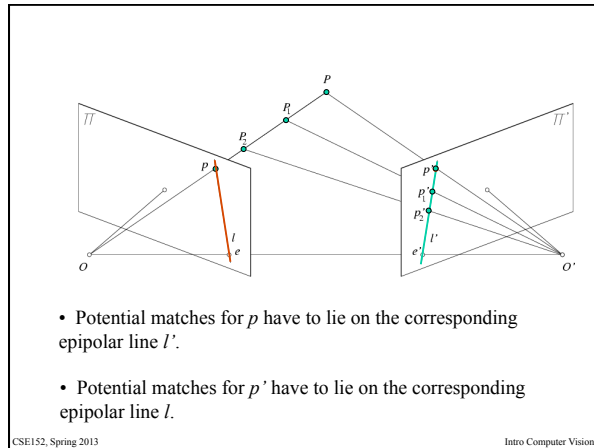
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## Random Dot Stereograms

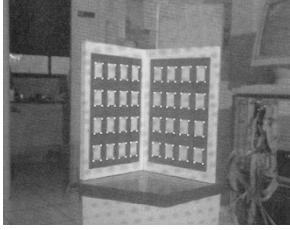


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## Calibration-Based Method



1. From image of known calibration fixture, determine intrinsic parameters and extrinsic relation of two cameras.
2. Compute the relative position and orientation of the two cameras from  $R_1, R_2, t_1, t_2$
3. Compute the Essential Matrix

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## The Eight-Point Algorithm (Longuet-Higgins, 1981)

$$p^T \mathcal{E} p' = 0 \quad \text{with} \quad \mathcal{E} = [t_x] \mathcal{R}$$

$$[u, v, 1] \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0 \quad \Rightarrow \quad [uu' \quad uv' \quad u \quad vu' \quad vv' \quad v \quad u' \quad v' \quad 1] \begin{bmatrix} E_{11} \\ E_{12} \\ E_{13} \\ E_{21} \\ E_{22} \\ E_{23} \\ E_{31} \\ E_{32} \\ E_{33} \end{bmatrix} = 0$$

• Set  $E_{33}$  to 1  
• Use 8 points  $(u_i, v_i), i=1..8$

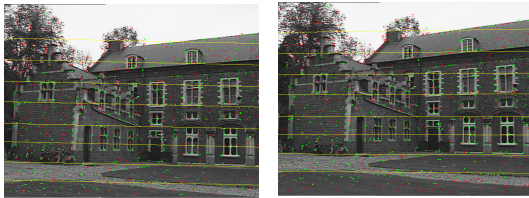
$$\begin{bmatrix} u_1 u'_1 & u_1 v'_1 & u_1 & v_1 u'_1 & v_1 v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 u'_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 & 1 \\ u_3 u'_3 & u_3 v'_3 & u_3 & v_3 u'_3 & v_3 v'_3 & v_3 & u'_3 & v'_3 & 1 \\ u_4 u'_4 & u_4 v'_4 & u_4 & v_4 u'_4 & v_4 v'_4 & v_4 & u'_4 & v'_4 & 1 \\ u_5 u'_5 & u_5 v'_5 & u_5 & v_5 u'_5 & v_5 v'_5 & v_5 & u'_5 & v'_5 & 1 \\ u_6 u'_6 & u_6 v'_6 & u_6 & v_6 u'_6 & v_6 v'_6 & v_6 & u'_6 & v'_6 & 1 \\ u_7 u'_7 & u_7 v'_7 & u_7 & v_7 u'_7 & v_7 v'_7 & v_7 & u'_7 & v'_7 & 1 \\ u_8 u'_8 & u_8 v'_8 & u_8 & v_8 u'_8 & v_8 v'_8 & v_8 & u'_8 & v'_8 & 1 \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{12} \\ E_{13} \\ E_{21} \\ E_{22} \\ E_{23} \\ E_{31} \\ E_{32} \\ E_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Solve  $E_{11}$  to  $E_{32}$  --  
The Essential Matrix

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## Epipolar geometry example



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## The Eight-Point Algorithm (Longuet-Higgins, 1981)

$$[u, v, 1] \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0 \quad \Rightarrow \quad [uu' \quad uv' \quad u \quad vu' \quad vv' \quad v \quad u' \quad v' \quad 1] \begin{bmatrix} E_{11} \\ E_{12} \\ E_{13} \\ E_{21} \\ E_{22} \\ E_{23} \\ E_{31} \\ E_{32} \\ E_{33} \end{bmatrix} = 0$$

- Following, the text, view this as system of homogenous equations in  $E_{11}$  to  $E_{33}$
- Solve as Eigenvector corresponding to the smallest Eigenvalue of matrix created from the image data.

Equivalent to solving

Minimize:

$$\sum_{i=1}^n (p_i^T \mathcal{E} p_i)^2$$

under the constraint

$$|\mathcal{E}|^2 = 1.$$

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## The Fundamental Matrix

The epipolar constraint is given by:  $p^T \mathcal{E} p' = 0$  with  $\mathcal{E} = [t_x] \mathcal{R}$  where  $p$  and  $p'$  are 3-D coordinates of the image coordinates of points in the two images.

Without calibration, we can still identify corresponding points in two images, but we can't convert to 3-D coordinates. However, the relationship between the calibrated coordinates  $(p, p')$  and uncalibrated image coordinates  $(q, q')$  can be expressed as  $p = Aq$ , and  $p' = A'q'$

Therefore, we can express the epipolar constraint as:

$$(Aq)^T E (A'q') = q^T (A^T E A') q' = q^T F q' = 0$$

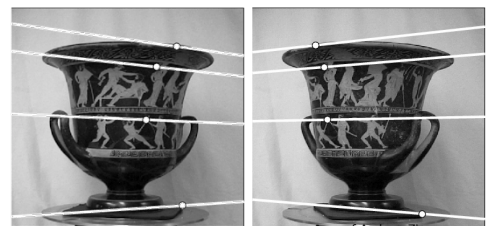
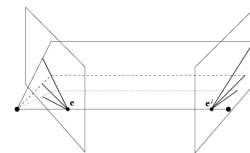
where  $F$  is called the Fundamental Matrix.

Can estimate  $F$  using 8 point algorithm WITHOUT CALIBRATION

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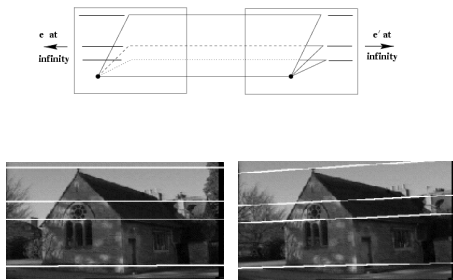
## Example: converging cameras



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courtesy of Andrew Zisserman

### Example: motion parallel with image plane

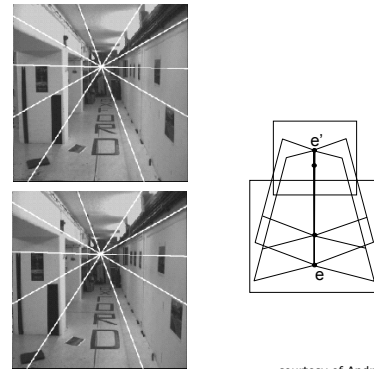


(simple for stereo → rectification)

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courtesy of Andrew Zisserman

### Example: forward motion



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courtesy of Andrew Zisserman

### Properties of the Essential Matrix

$$\mathbf{p}^T \mathbf{E} \mathbf{p}' = 0 \quad \text{with} \quad \mathbf{E} = [\mathbf{t}_\times] \mathbf{R}$$

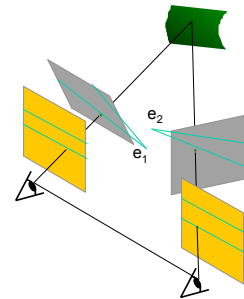
- $\mathbf{E} \mathbf{p}'$  is the epipolar line associated with  $\mathbf{p}'$ .
- $\mathbf{E}^T \mathbf{p}$  is the epipolar line associated with  $\mathbf{p}$ .
- $\mathbf{E}$  is singular because  $[\mathbf{t}_\times]$  is singular and the product two singular matrices is singular
- $\mathbf{E} \mathbf{e}' = 0$  and  $\mathbf{E}^T \mathbf{e} = 0$ .  
Because  $\mathbf{E}$  is singular, one of the Eigenvalues is zero. The Eigenvector corresponding to the zero Eigenvalue is an epipole. The same holds for  $\mathbf{E}^T$ .

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### Rectification

Given a pair of images, transform both images so that epipolar lines are scan lines.

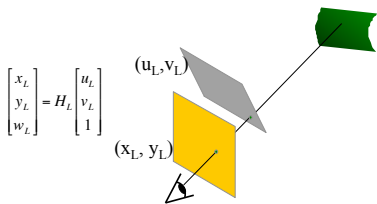


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### Rectification

Under perspective projection, the mapping from a plane to a plane is given by a projective transformation (aka homography).

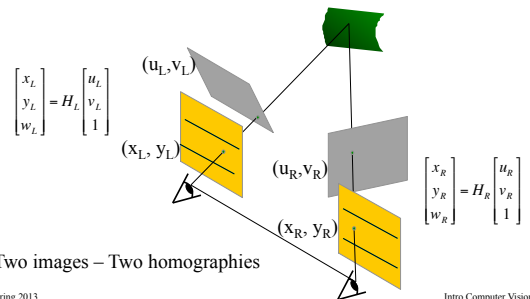


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### Rectification

Under perspective projection, the mapping from a plane to a plane is given by a projective transformation (aka homography).



Two images – Two homographies

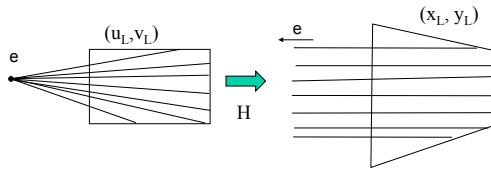
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### Image pair rectification

Simplify stereo matching by warping the images

Apply projective transformation so that epipolar lines correspond to horizontal scanlines



H should map epipole  $e$  to  $(1, 0, 0)$ , a point at infinity  
H should minimize image distortion

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = H e$$

Note that rectified images usually not rectangular  
See Section 7.3.7 for specific method

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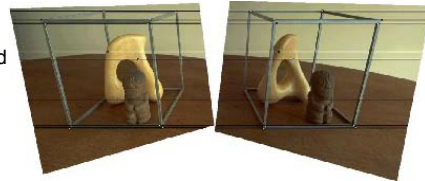
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## Rectification

Input  
Image  
Pair



Rectified  
Image  
Pair



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