CSE252B – Computer Vision II – Final Exam

Instructor: Prof. Serge Belongie.

`http://cseweb.ucsd.edu/classes/wi12/cse252B-a/`

3:00pm-6:00pm Mon. Mar. 19, 2012.

On this exam you are allowed to use a calculator and two 8.5” by 11” sheets of notes. The total number of points possible is 40. In order to get full credit you must show all your work. Good luck!

1. (2 pts.) Referring to the expression for a 3D point imaged by a general camera, i.e., $\lambda \boldsymbol{x} = \Pi \boldsymbol{X}$, with $\Pi = K[R, \boldsymbol{T}]$, explain why distant points (e.g., on the moon or a mountain) appear stationary when viewed from a translating vehicle.

2. Let $\boldsymbol{l} = (0, 0, 1)^\top$ denote the homogeneous coordinates of a line in $\mathbb{P}^2$ and let $C = \mathrm{diag}\{1, 1, -1\}$ be the coefficient matrix for the conic $\boldsymbol{x}^\top C \boldsymbol{x} = 0$.

   (a) (1 pt.) What is the special name for $\boldsymbol{l}$?

   (b) (1 pt.) What do you get if you intersect $\boldsymbol{l}$ and C?

3. Suppose you are designing a system for automatic projective distortion correction (also known as “keystone correction”) for an LCD projector. In this problem, assume you are able to extract the lines forming the bounding quadrilateral of the image on the projection screen.

   (a) (2 pts.) Explain how to identify the image of the line at infinity (call it $\boldsymbol{l} = (a, b, c)^\top$).

   (b) (1 pt.) Give the entries of the matrix $H \in GL(3)$ needed to perform an upgrade from projective to affine.

   (c) (1 pt.) In general, what will the shape of the upgraded image be?

   (d) (1 pt.) What approach could you use to upgrade directly from projective to Euclidean?

4. Essential matrix.

   (a) (1 pt.) How many degrees of freedom does E have?

   (b) (3 pts.) Explain where the degrees of freedom come from.

5. Both the Lucas-Kanade optical flow method and the Förstner operator require the computation of a special 2×2 symmetric matrix in a window around each pixel as an intermediate step.

   (a) (3 pt.) What are the entries of this matrix?

   (b) (3 pts.) Prove that this matrix is positive semidefinite.

   (c) (1 pts.) For what type of image neighborhoods does this matrix have rank 1?

6. Consider the homogeneous transformation $H \in GL(2)$.

   (a) (1 pt) How many degrees of freedom does H have?

   (b) (1 pt) Given a set of points in $\mathbb{P}^1$, what does H represent?

   (c) (2 pts) How many point correspondences are needed to estimate H? What constitutes general position in this case?

   (d) (3 pts) Given 3 points in $\mathbb{P}^1$ in general position, find the mapping H that leaves the outer two points fixed while moving the inner point to any desired location.

7. The orthographic camera model.

   (a) (1 pt.) Describe the conditions under which the orthographic camera model is reasonable.

(b) (2 pt.) Define the "Hitchcock zoom" effect and explain how to produce it using a video camera with a zoom lens. You can assume you have a tripod with wheels, or a very steady hand.

(c) (1 pt.) Which SFM method did we study that takes as input a set of tracked points captured under orthographic projection?

8. (2 pt) What kind of transformation in $\mathbb{P}^2$ leaves the line at infinity $\boldsymbol{l}_\infty$ unchanged? Apply this transformation to $\boldsymbol{l}_\infty$ and show that it is not affected.

9. (2 pts) The epipolar rectification algorithm returns two 2D homographies, H_1 and H_2, which are applied to image 1 and image 2, respectively. What equivalent 3D transformation can be applied to camera 1 and camera 2 to produce the same effect?

10. Suppose you capture two frames by rotating a camera about its optical center.

    (a) (1 pt.) Can you use the four point algorithm to estimate H in this case?

    (b) (1 pt.) If yes, explain a practical use for H. If no, explain how the motion would have to change to make this possible.

11. (1 pt) When estimating the Fundamental matrix from noisy data we set its third singular value to zero. Let F and F' denote the Fundamental matrix before and after this operation. How are the epipolar lines produced by F different from those produced by F'?

12. Consider the two lines $y = x$ and $y = x + 1$.

    (a) (1 pt.) Write down the expression for each line ($\boldsymbol{l}_1$ and $\boldsymbol{l}_2$) in homogeneous coordinates.

    (b) (1 pt.) Solve for their point of intersection.