CSE252B – Computer Vision II – Final Exam
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`http://www-cse.ucsd.edu/classes/wi11/cse252b`
1:30pm-4:30pm Tue. Mar. 15, 2011.

On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 50. In order to get full credit you must show all your work. Good luck!

1. Let $\boldsymbol{l} = (0, 0, 1)^\top$ denote the homogeneous coordinates of a line in $\mathbb{P}^2$ and let $C = \mathrm{diag}\{1, 1, -1\}$ be the coefficient matrix for the conic $\boldsymbol{x}^\top C \boldsymbol{x} = 0$.
   (a) (1 pt.) What is the special name for $\boldsymbol{l}$?
   (b) (1 pt.) What do you get if you intersect $\boldsymbol{l}$ and C?
2. (1 pt) When estimating the Fundamental matrix from noisy data we set its third singular value to zero. Let F and F' denote the Fundamental matrix before and after this operation. How are the epipolar lines produced by F different from those produced by F'?
3. Consider an image of the chalkboard in Warren Lecture Hall 2204 captured by a student sitting in class. Let $\boldsymbol{x}_1 \in \mathbb{P}^2$ denote the homogeneous coordinates of a point on the chalkboard, and let $\boldsymbol{x}_2 \in \mathbb{P}^2$ denote its image.
   (a) (1 pt) What general class of transformation $\mathcal{T}(\cdot)$ maps $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$?
   (b) (2 pts) How many corresponding point pairs are needed to estimate $\mathcal{T}(\cdot)$? What are the conditions on the coordinates of these points for the solution to be valid?
   (c) (1 pt) Suppose the student is seated very far away from the chalkboard. What simplified class of global transformation can be used to approximate $\mathcal{T}(\cdot)$?
   (d) (2 pts) Now suppose the student is seated at an arbitrary location. The professor marks a point $\boldsymbol{x}_1^*$ on the chalkboard. Describe how you would compute a linear approximation $\widetilde{\mathcal{T}}(\cdot)$ to $\mathcal{T}(\cdot)$ that holds in the vicinity of $\boldsymbol{x}_1^*$. What class of transformation is $\widetilde{\mathcal{T}}(\cdot)$?
   (e) (3 bonus pts) Solve for $\widetilde{\mathcal{T}}(\cdot)$ in terms of the coefficients of $\mathcal{T}(\cdot)$.
4. Hartley normalization.
   (a) (1 pt.) Explain the motivation behind Hartley normalization.
   (b) (2 pts.) If we regard the Hartley normalization matrix H as a guess of the calibration matrix K, what assumptions is it making about the calibration parameters?
5. (2 pts) The epipolar rectification algorithm returns two 2D homographies, H_1 and H_2, which are applied to image 1 and image 2, respectively. What equivalent 3D transformation can be applied to camera 1 and camera 2 to produce the same effect?
6. Many natural and man-made objects (e.g., airplanes) exhibit bilateral symmetry. Suppose you capture a *single* image $I(x, y)$ of an airplane using a camera with unknown relative pose $(R, \boldsymbol{T})$ with respect to the coordinate frame of the airplane.
   (a) (4 pts) Given only $I(x, y)$ as input, explain how to estimate the 3D structure of the airplane using techniques from this course. This will only be possible up to a certain unknown transformation; name the class of that transformation.
   (b) (2 pts) The quality of the 3D reconstruction will depend on the relative camera pose. What are the worst choices of $(R, \boldsymbol{T})$? What are the best?

7. Write down an example of a conic matrix C corresponding to each of the following cases and solve for the points of intersection with $\boldsymbol{l}_\infty$.
   (a) (2 pts.) Ellipse.
   (b) (2 pts.) Parabola.
   (c) (2 pts.) Hyperbola.
8. Consider the two lines $y = 2x + a$ and $y = 2x - b$, where $a, b \in \mathbb{R}^+$.
   (a) (1 pt.) Write down the expression for each line ($\boldsymbol{l}_1$ and $\boldsymbol{l}_2$) in homogeneous coordinates.
   (b) (1 pt.) Solve for their point of intersection.
9. Consider the homogeneous transformation $H \in GL(2)$.
   (a) (1 pt) How many degrees of freedom does H have?
   (b) (1 pt) Given a set of points in $\mathbb{P}^1$, what does H represent?
   (c) (2 pts) How many ground truth coordinates are needed to estimate H? What condition must these points satisfy?
   (d) (3 pts) Given 3 points in $\mathbb{P}^1$ in general position, show that there exists a mapping H that leaves the outer two points fixed while moving the inner point to any desired location.
10. Consider a pair of images of a Rubik's Cube captured using an uncalibrated stereo rig.
    (a) (1 pt) Without making any assumptions about the Rubik's Cube, what can we determine about its 3D structure?
    (b) (2 pts) Suppose you determine the three vanishing points in the two images. What do they allow you to compute, and how is it useful?
    (c) (3 pts) Once the epipolar geometry is known, one could estimate the vanishing points in each image and solve for their projective depths. H&Z advise against this, however. Why is it not a good idea? What do they suggest as an alternative?
    (d) (2 pts) How many ground truth points are needed to perform a Euclidean upgrade? Make a sketch to indicate acceptable locations for this set of points.
11. Corner detection.
    (a) (2 pt.) Describe two situations in which a corner detected by the Förstner interest point operator does not correspond to a physical corner in the 3D scene.
    (b) (1 pt.) What problem do these 'false corners' present?
    (c) (1 pt.) Name a practical tool that can be used to get around this problem.
12. Suppose you capture two images related by a pure translation in the Z direction, e.g., forward motion through a hallway.
    (a) (1 pt.) Where is the epipole in this case?
    (b) (1 pt.) What problem arises if you try to apply standard epipolar rectification to this pair of images?
    (c) (1 pt.) Suggest a high-level approach to address this problem.