

Image Formation and Cameras (cont.)

Biometrics
CSE 190
Lecture 6

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Biometrics

Announcements

- Project topics due today

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Image Formation: Outline

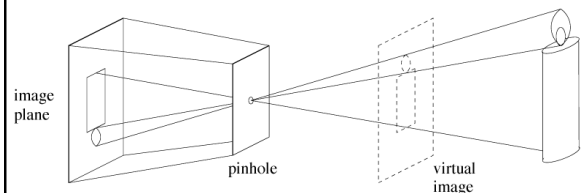
- Factors in producing images
- Projection
- Perspective
- Vanishing points
- Orthographic
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance

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Pinhole Camera: **Perspective projection**

- Abstract camera model - box with a small hole in it

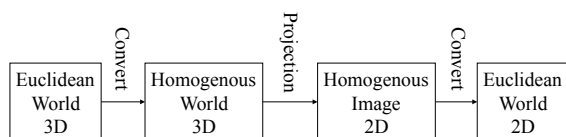


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Homogenous coordinates

- Our usual coordinate system is called a Euclidean or affine coordinate system
- Rotations, translations and projection in Homogenous coordinates can be expressed linearly as matrix multiplies



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Euclidean -> Homogenous-> Euclidean

In 2-D

- Euclidean -> Homogenous: $(x, y) \rightarrow k(x, y, 1)$
- Homogenous -> Euclidean: $(u, v, w) \rightarrow (u/w, v/w)$

In 3-D

- Euclidean -> Homogenous: $(x, y, z) \rightarrow k(x, y, z, 1)$
- Homogenous -> Euclidean: $(x, y, z, w) \rightarrow (x/w, y/w, z/w)$

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The equation of projection

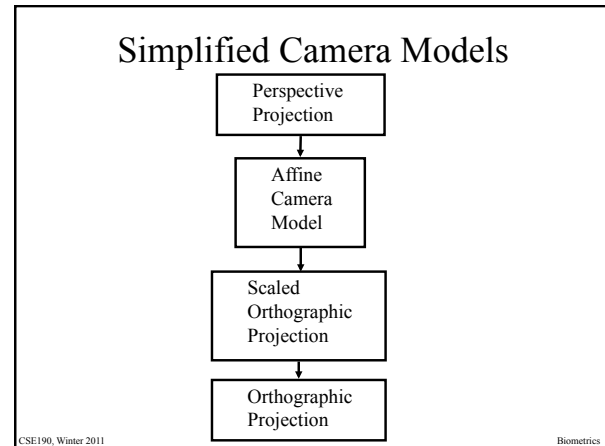
Homogenous Coordinates and Camera matrix

Cartesian coordinates:

$$(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z} \right)$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

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Affine Camera Model

Appropriate in Neighborhood About (x_0, y_0, z_0)

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point (x_0, y_0, z_0)).
- Drop terms of higher order than linear.
- Resulting expression is affine camera model

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- Perspective

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z} \begin{bmatrix} x \\ y \end{bmatrix}$$
- Assume that $f=1$, and perform a Taylor series expansion about (x_0, y_0, z_0)

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \frac{1}{z_0^2} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0) + \frac{1}{z_0} \begin{bmatrix} 1 \\ 0 \end{bmatrix} (x - x_0) + \frac{1}{z_0} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (y - y_0) + \frac{1}{2} \frac{2}{z_0^3} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0)^2 + \dots$$
- Dropping higher order terms and regrouping.

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\ 0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}\mathbf{p} + \mathbf{b}$$

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$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\ 0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}\mathbf{p} + \mathbf{b}$$

Rewrite Affine camera model in terms of Homogenous Coordinates

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0/z_0 \\ 0 & 1/z_0 & -y_0/z_0^2 & y_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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Scaled Orthographic projection

Starting with Affine camera mode
Take Taylor series about $(0, 0, z_0)$ – a point on optical axis

Euclidean Coordinates

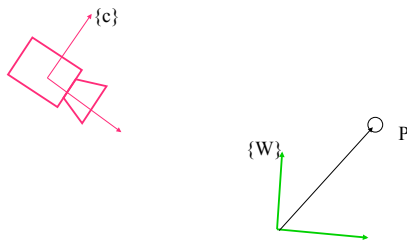
$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogenous Coordinates

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1/z_0 & 0 & 0 & 0 \\ 0 & 1/z_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

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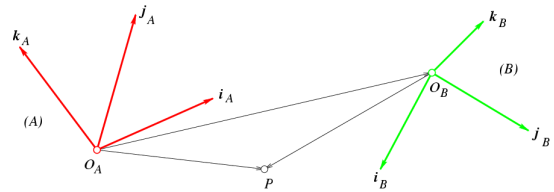
What if camera coordinate system differs from object coordinate system



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Coordinate Changes: Rigid Transformations



$${}^B P = {}^B R_A {}^A P + {}^B O_A$$

Rotation Matrix

Translation vector

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A rotation matrix R has the following properties:

- Its inverse is equal to its transpose $R^{-1} = R^T$
- Its determinant is equal to 1: $\det(R) = 1$.

Or equivalently:

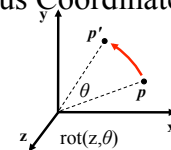
- Rows (or columns) of R form a right-handed orthonormal coordinate system.

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Rotation: Homogenous Coordinates

- About z axis



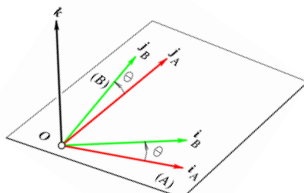
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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Roll-Pitch-Yaw

$$R = \text{rot}(\hat{i}, \alpha) \text{rot}(\hat{j}, \beta) \text{rot}(\hat{k}, \varphi)$$



Euler Angles

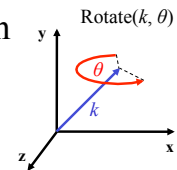
$$R = \text{rot}(\hat{k}', \alpha) \text{rot}(\hat{j}', \beta) \text{rot}(\hat{k}, \varphi)$$

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Rotation

- About (k_x, k_y, k_z) , a unit vector on an arbitrary axis (Rodrigues Formula)



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} k_x k_x (1-c) + c & k_z k_x (1-c) - k_y s & k_y k_x (1-c) + k_z s & 0 \\ k_y k_x (1-c) + k_z s & k_z k_x (1-c) + c & k_y k_z (1-c) - k_x s & 0 \\ k_z k_x (1-c) - k_y s & k_z k_x (1-c) - k_x s & k_z k_z (1-c) + c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

where $c = \cos \theta$ & $s = \sin \theta$

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Block Matrix Multiplication

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

What is AB ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Homogeneous Representation of Rigid Transformations

$$\begin{bmatrix} {}^B P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^B R & {}^B O_A \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} {}^A P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^B R & {}^B O_A \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} {}^A P \\ 1 \end{bmatrix} = {}^B T_A \begin{bmatrix} {}^A P \\ 1 \end{bmatrix}$$

Transformation represented by 4 by 4 Matrix

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Camera parameters

- Issue
 - camera may not be at the origin, looking down the z-axis
 - extrinsic parameters (Rigid Transformation)
 - one unit in camera coordinates may not be the same as one unit in world coordinates
 - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

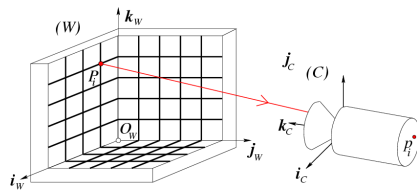
$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

3 x 3 4 x 4

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Camera Calibration



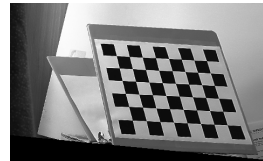
Given n points $P_1, \dots, P_n$ with *known* positions and their images $p_1, \dots, p_n$, estimate intrinsic and extrinsic camera parameters

- See Camera Calibration Toolbox for Matlab (Bouguet) http://www.vision.caltech.edu/bouguetj/calib_doc/

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Perspective Projection of Plane: Homography



The mapping under perspective projection of a plane in the scene to the image plane is given by a homography.

$$Y = HX$$

Where Y and X are respectively the homogenous 2-D coordinates of points on the plane and the homogenous 2-D image coordinates.

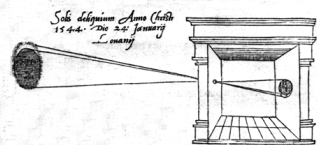
H is a 3x3 matrix.

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Camera Obscura

illum in tabula per radios Solis, quàm in celo contingit: hoc efficit in celo superior pars deliquit patitur, an radius apparuit inferior defecere, ut ratio exigit optica.



Sic not exatè Anno .1544. Lotus eclipsim Solis obseruauimus, inuenimusq; defecere paulò plus q̄ dex-

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

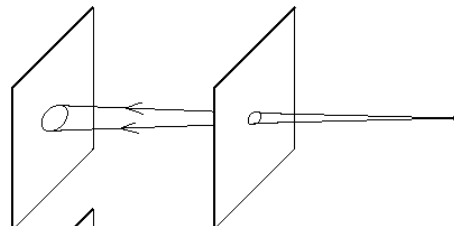
Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

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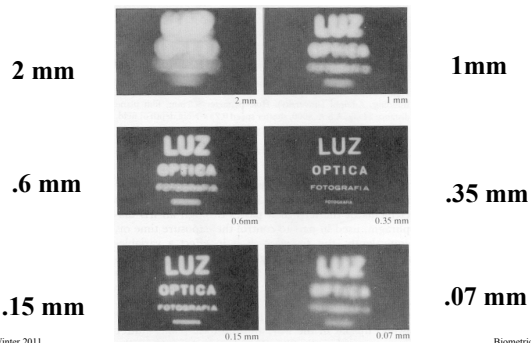
Getting more light – Bigger Aperture



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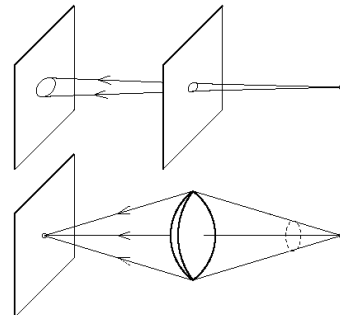
Pinhole Camera Images with Variable Aperture



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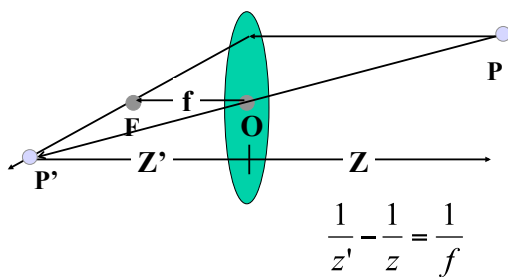
The reason for lenses



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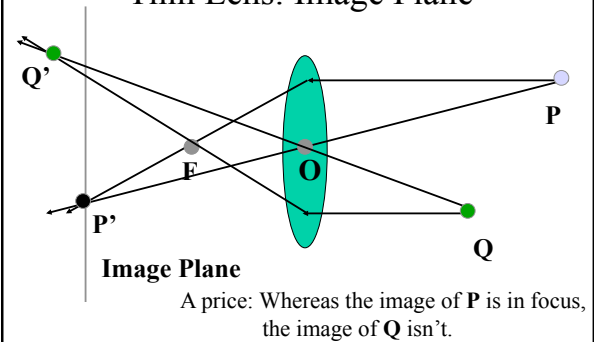
Thin Lens: Image of Point



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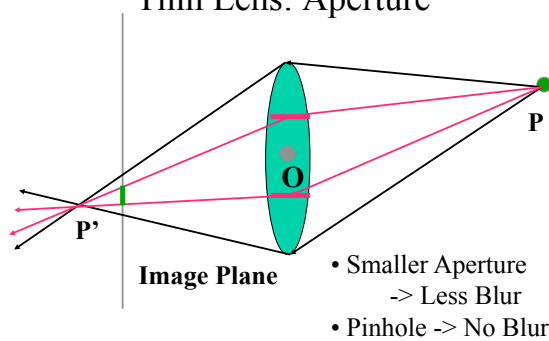
Thin Lens: Image Plane



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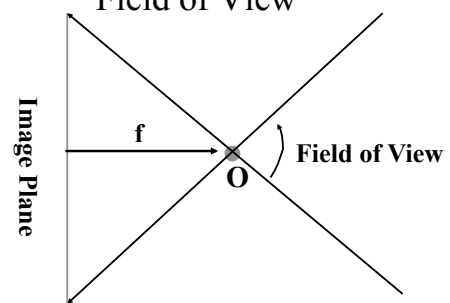
Thin Lens: Aperture



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Field of View



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Deviations from the lens model

Deviations from this ideal are **aberrations**

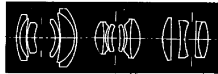
Two types

1. geometrical

- ☐ spherical aberration
- ☐ astigmatism
- ☐ distortion
- ☐ coma

2. chromatic

Aberrations are reduced by combining lenses



Compound lenses

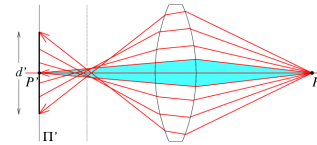
CSE252A, Fall 2009

Computer Vision I

Spherical aberration

rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lengths

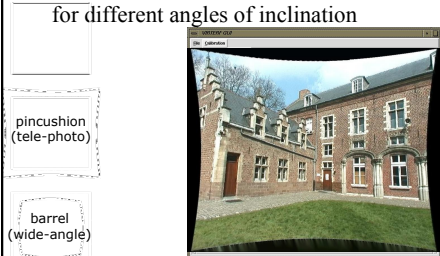


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Distortion

magnification/focal length different for different angles of inclination



Can be corrected! (if parameters are known)

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Chromatic aberration

Index of refraction of lens depends on wavelength of light

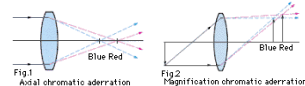


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Chromatic aberration

rays of different wavelengths focus in different planes



cannot be removed completely

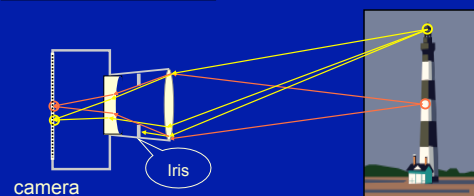
sometimes **achromatization** is achieved for more than 2 wavelengths



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Vignetting: Spatial Non-Uniformity



Litvinov & Schechner, radiometric nonidealities

Vignetting



- Only part of the light reaches the sensor
- Periphery of the image is dimmer

CSE252A, Fall 2009

Computer Vision I

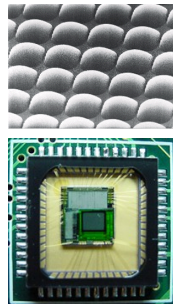
Image Brightness

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How Cameras Produce Images

- Basic process:
 - photons hit a detector
 - the detector becomes charged
 - the charge is read out as brightness
- Sensor types:
 - CCD (charge-coupled device)
 - high sensitivity
 - high power
 - cannot be individually addressed
 - blooming
 - CMOS
 - most common
 - simple to fabricate (cheap)
 - lower sensitivity, lower power
 - can be individually addressed



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Camera's sensor

- Measured pixel intensity is a function of irradiance integrated over
 - pixel's area
 - over a range of wavelengths
 - For some time

$$I = \iiint_{t \lambda x y} E(x, y, \lambda, t) s(x, y) q(\lambda) dy dx d\lambda dt$$

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Light at surfaces

Many effects when light strikes a surface -- could be:

- transmitted
 - Skin, glass
- reflected
 - mirror
- scattered
 - milk
- travel along the surface and leave at some other point
- absorbed
 - sweaty skin

Assume that

- surfaces don't fluoresce
 - e.g. scorpions, detergents
- surfaces don't emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

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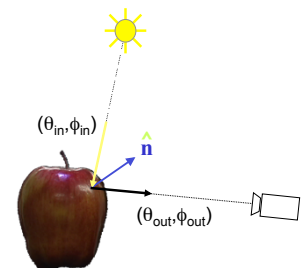
BRDF

- Bi-directional Reflectance Distribution Function

$$\rho(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out})$$

- Function of
 - Incoming light direction: θ_{in}, ϕ_{in}
 - Outgoing light direction: θ_{out}, ϕ_{out}

- Ratio of incident irradiance to emitted radiance



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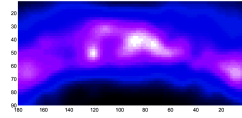
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Surface Reflectance Models

Common Models

- Lambertian
- Phong
- Physics-based
 - Specular [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
 - Diffuse [Hanrahan, Kreuger 1993]
 - Generalized Lambertian [Oren, Nayar 1995]
 - Thoroughly Pitted Surfaces [Koenderink et al 1999]
- Phenomenological [Koenderink, Van Doorn 1996]

Arbitrary Reflectance

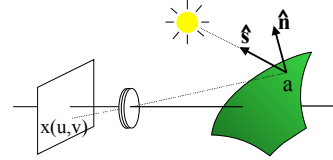


- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement [Dana et al, 1999], [Marschner]

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Lambertian Surface



At image location (u,v) , the intensity of a pixel $x(u,v)$ is:

$$x(u,v) = [a(u,v)\hat{n}(u,v)] \cdot [s_0\hat{s}] = b(u,v) \cdot s$$

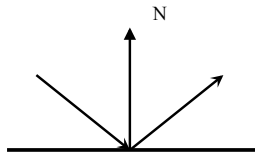
where

- $a(u,v)$ is the albedo of the surface projecting to (u,v) .
- $\hat{n}(u,v)$ is the direction of the surface normal.
- s_0 is the light source intensity.
- $\hat{s}$ is the direction to the light source.

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Specular Reflection: Smooth Surface

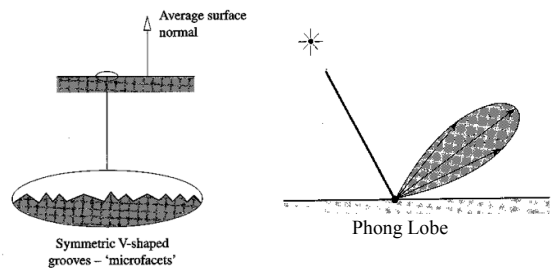


Phong – rough, specular

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Rough Specular Surface



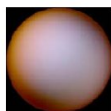
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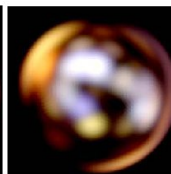
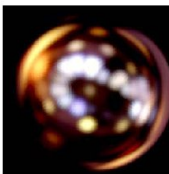
Phong Model



Mirror



Diffuse

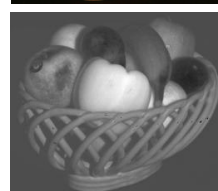


CS348B Lecture 10

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Pat Hanrahan, Spring 2002

SUV Color Space



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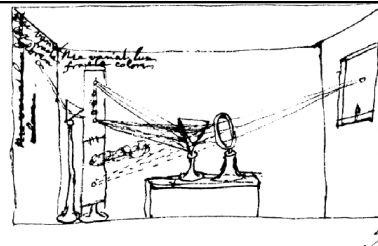
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Gloss removal



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4.1 NEWTON'S SUMMARY DRAWING of his experiments with light. Using a point source of light and a prism, Newton separated sunlight into its fundamental components. By reconverging the rays, he also showed that the decomposition is reversible.

From Foundations of Vision, Brian Wandell, 1995, via B. Freeman slides



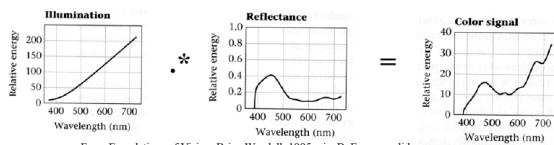
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Color Reflectance



Measured color spectrum is a function of the spectrum of the illumination and reflectance

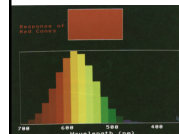


From Foundations of Vision, Brian Wandell, 1995, via B. Freeman slides

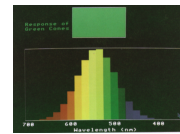
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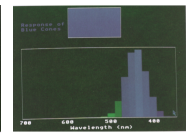
Color receptors



"Red" cone



"Green" cone



"Blue" cone

Response of k'th cone =
$$\int \rho_k(\lambda) E(\lambda) d\lambda$$

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Color Cameras

Eye:

Three types of Cones

Cameras:

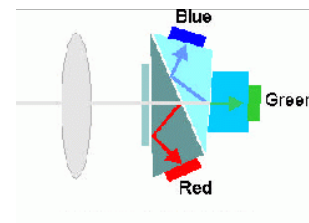
1. Prism (with 3 sensors)
 2. Filter mosaic
 3. Filter wheel
- ... and X3

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Prism color camera

Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation

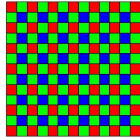
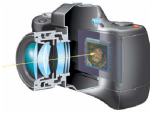


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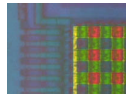
Biometrics

Filter mosaic

Coat filter directly on sensor



Bayer filter



Demosaicing (obtain full colour & full resolution image)

