



Pattern classification & Image Formation

Biometrics
CSE 190
Lecture 5

CSE190d, Winter 2011

Announcements

- Project Meetings –CSE Rm. 4120
- HW1: Not yet posted

CSE190d, Winter 2011

Discriminant Functions for the Normal Density

3

- We saw that the minimum error-rate classification can be achieved by the discriminant function

$$g_i(x) = \ln P(x | \omega_i) + \ln P(\omega_i)$$

- Case of multivariate normal

$$g_i(x) = -\frac{1}{2}(x - \mu_i)' \Sigma_i^{-1}(x - \mu_i) - \frac{d}{2} \ln 2\pi - \frac{1}{2} \ln |\Sigma_i| + \ln P(\omega_i)$$

- Decision boundary $g_i(x) = g_j(x)$

Pattern Classification, Chapter 2 (Part 3)

CASE I

4

Case $\Sigma_i = \sigma^2 I$ (I stands for the identity matrix)

$$g_i(x) = w_i' x + w_{i0} \text{ (linear discriminant function)}$$

where:

$$w_i = \frac{\mu_i}{\sigma^2}; \quad w_{i0} = -\frac{1}{2\sigma^2} \mu_i' \mu_i + \ln P(\omega_i)$$

(w_{i0} is called the threshold for the i th category!)

Pattern Classification, Chapter 2 (Part 3)

- The decision surfaces for a linear machine are pieces of **hyperplanes** defined by:

$$g_i(x) = g_j(x)$$

Pattern Classification, Chapter 2 (Part 3)

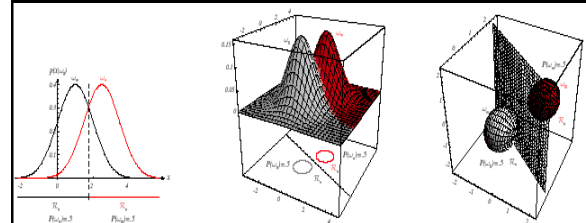


FIGURE 2.10. If the covariance matrices for two distributions are equal and proportional to the identity matrix, then the distributions are spherical in d dimensions, and the boundary is a generalized hyperplane of $d - 1$ dimensions, perpendicular to the line separating the means. In these one-, two-, and three-dimensional examples, we indicate $p(x|\omega_i)$ and the boundaries for the case $P(\omega_1) = P(\omega_2)$. In the three-dimensional case, the grid plane separates R_1 from R_2 . From: Richard O. Duda, Peter E. Hart, and David G. Stork, *Pattern Classification*. Copyright © 2001 by John Wiley & Sons, Inc.

Pattern Classification, Chapter 2 (Part 3)

7

The hyperplane separating $\mathcal{R}_i$ and $\mathcal{R}_j$ are given by

$$\mathbf{w}^T (\mathbf{x} - \mathbf{x}_0) = 0$$

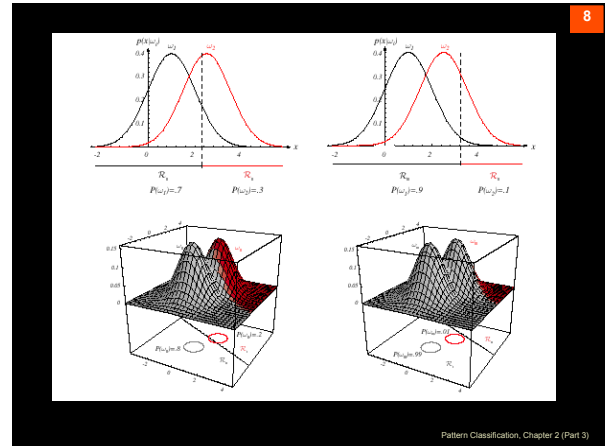
$$\mathbf{w} = \mu_i - \mu_j$$

$$\mathbf{x}_0 = \frac{1}{2}(\mu_i + \mu_j) - \frac{\sigma^2}{\|\mu_i - \mu_j\|^2} \ln \frac{P(\omega_i)}{P(\omega_j)} (\mu_i - \mu_j)$$

always orthogonal to the line linking the means!

if $P(\omega_i) = P(\omega_j)$ then $\mathbf{x}_0 = \frac{1}{2}(\mu_i + \mu_j)$

Pattern Classification, Chapter 2 (Part 3)



Case II

9

Case $\Sigma_i = \Sigma$
(covariance of all classes are identical but arbitrary!)

$$g_i(x) = -\frac{1}{2}(x - \mu_i)^T \Sigma_i^{-1} (x - \mu_i) - \frac{d}{2} \ln |2\pi| - \frac{1}{2} \ln |\Sigma_i| + \ln P(\omega_i)$$

$$= -\frac{1}{2}(x - \mu_i)^T \Sigma^{-1} (x - \mu_i) - \frac{1}{2} \ln |\Sigma| + \ln P(\omega_i)$$

$$= -\frac{1}{2}(x - \mu_i)^T \Sigma^{-1} (x - \mu_i) + \ln P(\omega_i)$$

Now consider case where $P(\omega_i) = P(\omega_j)$,

$$g_i(x) = -(x - \mu_i)^T \Sigma^{-1} (x - \mu_i)$$

$(x - \mu_i)^T \Sigma^{-1} (x - \mu_i)$ is sometimes called the Mahalanobis distance

Pattern Classification, Chapter 2 (Part 3)

Case II

10

Case $\Sigma_i = \Sigma$ (covariance of all classes are identical but arbitrary!)

Hyperplane separating $\mathcal{R}_i$ and $\mathcal{R}_j$

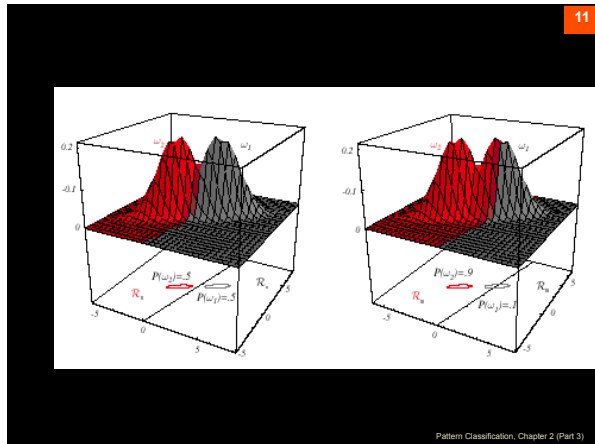
$$\mathbf{w}^T (\mathbf{x} - \mathbf{x}_0) = 0$$

$$\mathbf{w} = \Sigma^{-1} (\mu_i - \mu_j)$$

$$\mathbf{x}_0 = \frac{1}{2}(\mu_i + \mu_j) - \frac{\ln [P(\omega_i) / P(\omega_j)]}{(\mu_i - \mu_j)^T \Sigma^{-1} (\mu_i - \mu_j)} (\mu_i - \mu_j)$$

Here the hyperplane separating $\mathcal{R}_i$ and $\mathcal{R}_j$ is generally not orthogonal to the line between the means!

Pattern Classification, Chapter 2 (Part 3)



CASE III

12

Case $\Sigma_i = \text{arbitrary}$

The covariance matrices are different for each category

$$\mathbf{g}_i(\mathbf{x}) = \mathbf{x}^T \mathbf{W}_i \mathbf{x} + \mathbf{w}_{i0}^T \mathbf{x} = \mathbf{w}_{i0}$$

where :

$$\mathbf{W}_i = -\frac{1}{2} \Sigma_i^{-1}$$

$$\mathbf{w}_i = \Sigma_i^{-1} \mu_i$$

$$\mathbf{w}_{i0} = -\frac{1}{2} \mu_i^T \Sigma_i^{-1} \mu_i - \frac{1}{2} \ln |\Sigma_i| + \ln P(\omega_i)$$

Here the separating surfaces are Hyperquadrics which are:
hyperplanes, pairs of hyperplanes, hyperspheres,
hyperellipsoids, hyperparaboloids, hyperhyperboloids)

Pattern Classification, Chapter 2 (Part 3)

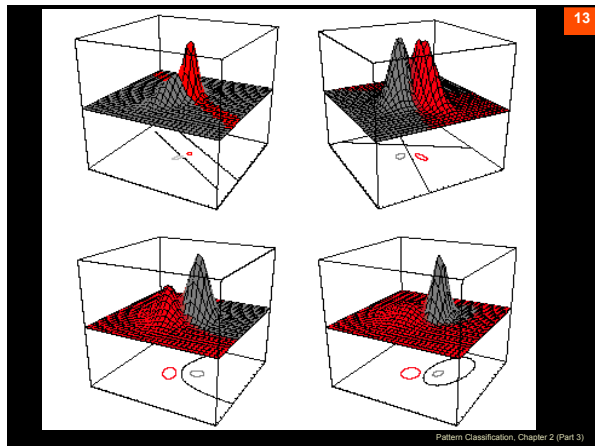


Image Formation: Outline

- Factors in producing images
- Projection
- Perspective
- Vanishing points
- Orthographic
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance

CSE190, Winter 2011

Biometrics

Earliest Surviving Photograph

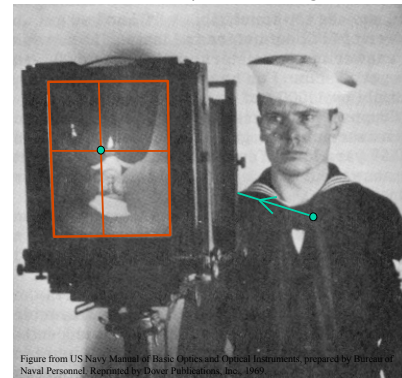


- First photograph on record, "la table service" by Nicéphore Niépce in 1822.
- Note: First photograph by Niépce was in 1816.

CSE190, Winter 2011

Biometrics

Images are two-dimensional patterns of brightness values.



CSE190, Winter 2011

They are formed by the projection of 3D objects.

Biometrics

Effect of Lighting: Monet



CSE190, Winter 2011

Biometrics

Change of Viewpoint: Monet



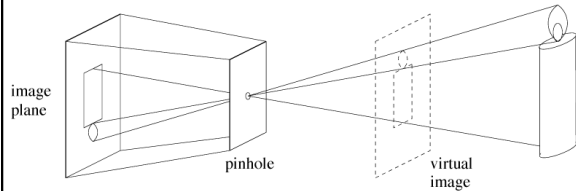
Haystack at Chailly at Sunrise (1865)

CSE190, Winter 2011

Biometrics

Pinhole Camera: **Perspective projection**

- Abstract camera model - box with a small hole in it

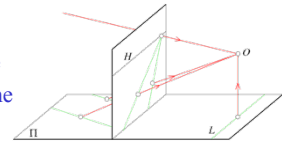


CSE190, Winter 2011 Forsyth&Ponce

Biometrics

Geometric properties of projection

- Points go to **points**
- Lines go to **lines**
- Planes go to **whole image or half-plane**
- Polygons go to **polygons**
- Angles & distances not preserved
- Degenerate cases:
 - line through focal point yields **point**
 - plane through focal point yields **line**

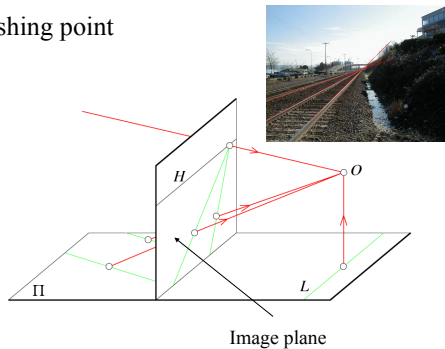


CSE190, Winter 2011

Biometrics

Parallel lines meet in the image

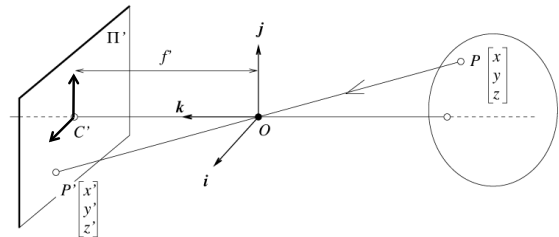
- vanishing point



CSE190, Winter 2011

Biometrics

Equation of Perspective Projection



Cartesian coordinates:

- We have, by similar triangles, that $(x, y, z) \rightarrow (f' x/z, f' y/z, f')$
- Establishing an image plane coordinate system at C' aligned with i and j , we get

$$(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})$$

CSE190

Computer Vision I

A Digression

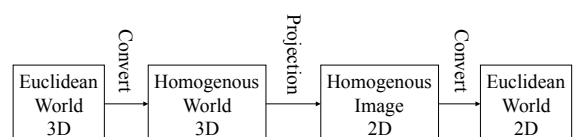
Homogenous Coordinates
and
Camera Matrices

CSE190, Winter 2011

Biometrics

Homogenous coordinates

- Our usual coordinate system is called a Euclidean or affine coordinate system
- Rotations, translations and projection in Homogenous coordinates can be expressed linearly as matrix multiplies



CSE190, Winter 2011

Biometrics

Homogenous coordinates

A way to represent points in a projective space

1. Add an extra coordinate
e.g., $(x,y) \rightarrow (x,y,1)=(u,v,w)$
 - Why do this?
 - Possible to represent points “at infinity”
 - Where parallel lines intersect
 - Where parallel planes intersect
 - Possible to write the action of a perspective camera as a matrix
2. Impose equivalence relation such that $(\lambda \neq 0)$
 $(u,v,w) \approx \lambda(u,v,w)$
i.e., $(x,y,1) \approx (\lambda x, \lambda y, \lambda)$
3. “Point at infinity” – zero for last coordinate
e.g., $(x,y,0)$

CSE190, Winter 2011

Biometrics

Euclidean $\rightarrow$ Homogenous $\rightarrow$ Euclidean

In 2-D

- Euclidean $\rightarrow$ Homogenous: $(x, y) \rightarrow k(x, y, 1)$
- Homogenous $\rightarrow$ Euclidean: $(u, v, w) \rightarrow (u/w, v/w)$

In 3-D

- Euclidean $\rightarrow$ Homogenous: $(x, y, z) \rightarrow k(x, y, z, 1)$
- Homogenous $\rightarrow$ Euclidean: $(x, y, z, w) \rightarrow (x/w, y/w, z/w)$

CSE190, Winter 2011

Biometrics

The camera matrix

$$(x, y, z) \rightarrow \left(f \frac{x}{z}, f \frac{y}{z} \right)$$

Turn this expression into homogenous coordinates

- HC’s for 3D point are (X, Y, Z, T)
- HC’s for point in image are (U, V, W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Perspective
Camera Matrix
A 3x4 matrix

CSE190, Winter 2011

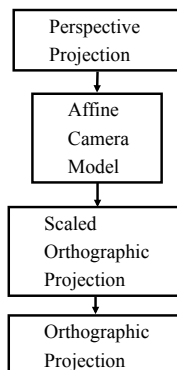
Biometrics

End of the Digression

CSE190, Winter 2011

Biometrics

Simplified Camera Models

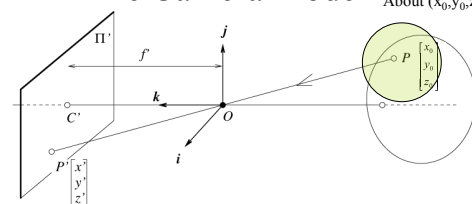


CSE190

Computer Vision I

Affine Camera Model

Appropriate in Neighborhood About (x_0, y_0, z_0)



- Take Perspective projection equation, and perform Taylor Series Expansion about (some point (x_0, y_0, z_0)).
- Drop terms of higher order than linear.
- Resulting expression is affine camera model

CSE190, Winter 2011

Biometrics

- Perspective

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{z} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Assume that $f=l$, and perform a Taylor series expansion about (x_0, y_0, z_0)

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \frac{1}{z_0^2} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0) + \frac{1}{z_0} \begin{bmatrix} 1 \\ 0 \end{bmatrix} (x - x_0) + \frac{1}{z_0} \begin{bmatrix} 0 \\ 1 \end{bmatrix} (y - y_0) + \frac{1}{2} \frac{2}{z_0^3} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} (z - z_0)^2 + \dots$$

- Dropping higher order terms and regrouping.

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\ 0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{Ap} + \mathbf{b}$$

CSE190, Winter 2011

Biometrics

$$\begin{bmatrix} u \\ v \end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\ 0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{Ap} + \mathbf{b}$$

Rewrite Affine camera model
in terms of Homogenous Coordinates

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} \approx \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0/z_0 \\ 0 & 1/z_0 & -y_0/z_0^2 & y_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

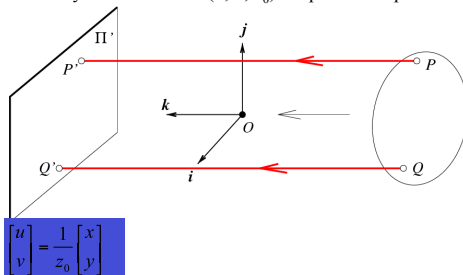
CSE190, Winter 2011

Biometrics

Orthographic projection

Starting with Affine camera mode

Take Taylor series about $(0, 0, z_0)$ – a point on optical axis



CSE190, Winter 2011

Biometrics

The projection matrix for orthographic projection

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 1/z_0 & 0 & 0 & 0 \\ 0 & 1/z_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

Parallel lines project to parallel lines
Ratios of distances are preserved under orthographic

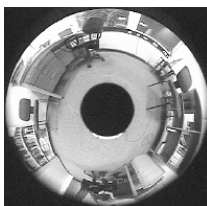
CSE190, Winter 2011

Biometrics

Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnicam (hemispherical)



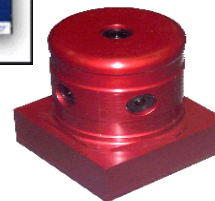
Light Probe (spherical)



CSE190, Winter 2011

Biometrics

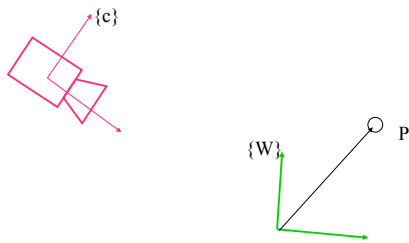
Some Alternative “Cameras”



CSE190, Winter 2011

Biometrics

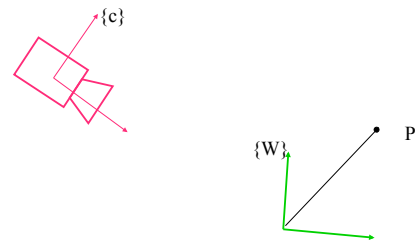
What if camera coordinate system differs from object coordinate system



CSE190, Winter 2011

Biometrics

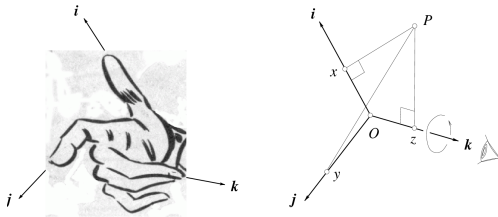
What if camera coordinate system differs from object coordinate system



CSE190

Computer Vision I

Euclidean Coordinate Systems

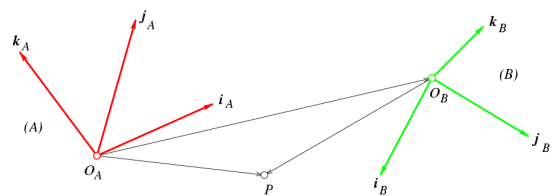


$$\begin{cases} x = \overline{OP} \cdot \mathbf{i} \\ y = \overline{OP} \cdot \mathbf{j} \\ z = \overline{OP} \cdot \mathbf{k} \end{cases} \Leftrightarrow \overline{OP} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Leftrightarrow \mathbf{P} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

CSE190, Winter 2011

Biometrics

Coordinate Changes: Rigid Transformations



$${}^B P = {}^B R^A P + {}^B O_A$$

Rotation Matrix

Translation vector

CSE190, Winter 2011

Biometrics