

BIOMETRICS
CSE190
Winter 2011

Assignment 1

Due: January 27, 2011

1. For a single feature, what are the optimal decision regions where:

- $\Omega = \{\omega_1, \omega_2\}$
- $P(x | \omega_1) = N(4, 1)$ (Normal distribution)
- $P(x | \omega_2) = N(3, 0.4)$
- $P(\omega_1) = 2/3$
- $P(\omega_2) = 1/3$
- $\lambda_{11} = 0, \lambda_{12} = 1, \lambda_{21} = 1, \lambda_{22} = 0$

2. Select the optimal decision regions for the above problem when:

- $\lambda_{11} = 1, \lambda_{12} = 4, \lambda_{21} = 3, \lambda_{22} = 2$

3. *Problem 2.13 in DHS.* In many pattern classification problems one has the option either to assign the pattern to one of c classes, or to reject it as being unrecognizable. If the cost for rejects is not too high, rejection may be a desirable action. Let

$$\lambda(\alpha_i | \omega_j) = \begin{cases} 0 & i = j, i, j = 1, \dots, c \\ \lambda_r & i = c + 1 \\ \lambda_s & \text{Otherwise} \end{cases}$$

Where λ_r is the loss incurred for choosing the $(c+1)$ th action, rejection, and λ_s is the loss incurred for making any substitution error. Show that the minimum risk is obtained if we decide ω_i if $P(\omega_i | \mathbf{x}) \geq P(\omega_j | \mathbf{x})$ for all j , and if $P(\omega_i | \mathbf{x}) > 1 - \lambda_r / \lambda_s$, and reject otherwise. What happens if $\lambda_r = 0$? What happens if $\lambda_r > \lambda_s$?

4.. Consider the classification problem with rejection option.

(a) Use the results from problem 2.13 (above) to show that the following discriminant functions are optimal for such problems:

$$g_i(\mathbf{x}) = \begin{cases} P(\mathbf{x} | \omega_i) P(\omega_i) & i = 1, \dots, c \\ \frac{\lambda_s - \lambda_r}{\lambda_s} \sum_{j=1}^c P(\mathbf{x} | \omega_j) P(\omega_j) & i = c + 1 \end{cases}$$

(b) Plot these discriminant functions and decisions regions for the two category, 1-D case having

$$P(x | \omega_1) = N(1, 1)$$

$$P(x | \omega_2) = N(-1, 1)$$

$$P(\omega_1) = P(\omega_2) = 1/2$$

$$\lambda_r/\lambda_s = 1/4$$

(c) Describe qualitatively what happens as λ_r/λ_s is increased from 0 to 1.

(d) Repeat (b) and (c) for the case with

$$P(x | \omega_1) = N(1, 1)$$

$$P(x | \omega_2) = N(0, 1/4)$$

$$P(\omega_1) = 1/3, P(\omega_2) = 2/3$$

$$\lambda_r/\lambda_s = 1/2$$

5. Suppose we have two normal distributions with the same covariances but different means: $N(\mu_1, \Sigma)$ and $N(\mu_2, \Sigma)$. In terms of their prior probabilities, $P(\omega_1)$ and $P(\omega_2)$, state the condition that the Bayes decision boundary does **NOT** pass between the two means.