

## CSE20 Exercise 1, February 12, 2010,

1. Residual Number System: Represent 38 with a residual number system of moduli  $(m_1, m_2, m_3) = (3, 5, 7)$ .
2. Residual Number System: Suppose  $(x\%5, x\%7, x\%11) = (1, 2, 3)$ , where symbol  $\%$  denotes modulus operation. Find the smallest positive integer  $x$  that satisfies this system.
3. Residual Number System: Show the operation of  $38 + 44$  in a residual number system with moduli  $(m_1, m_2, m_3) = (3, 5, 7)$ .
4. Residual Number System: Show the operation of  $19 \times 15$  in a residual number system with moduli  $(m_1, m_2, m_3) = (5, 13, 14)$ .
5. Residual Number System: State and prove the Chinese remainder theorem.
6. Boolean Algebra: Prove that for any  $a$  and  $b$  in the set  $B$  of a Boolean algebra,  $(a + b)(a + b') = a$ .
7. Boolean Algebra: Prove general associativity holds for  $+$  in any Boolean algebra. For all  $n \geq 1$ ,

$$a_1 + (a_2 + (a_2 + (\dots + a_n))) = (((a_1 + a_2) + a_3) + \dots) + a_n$$

You may assume that associativity holds for  $n = 2$ .

8. Boolean Algebra: State and prove DeMorgan's laws.
9. Boolean Algebra: Show the operation tables for a Boolean algebra of four elements.
10. Boolean Algebra: Simplify formula  $(pq + r')(p + r)(q + r)$ .
11. Boolean Algebra: Express Boolean function  $E(x, y, z) = (x' + y)(xy)'(x + y' + z)$  in sum-of-products form.
12. Boolean Algebra: Express Boolean function  $E(x, y, z) = xy + (x + z)' + x'y'z$  in product-of-sums form.
13. Boolean Algebra: Prove or disprove the Boolean equation,  $(a'b' + c)(a + b)(b' + ac)' = a'bc$ .
14. Boolean Algebra: Reduce the following to an expression of a minimal number of literals (4).  $abc'd + ab'c + bc'd + ab'c' + acd + a'bcd$ .