Solution of CSE20 Exercise 1

January 27, 2010

1. $(1000110100)_2$

2. 53

3. $3^5 \times 4^4$, or $0 \sim (3^5 \times 4^4 - 1)$

4.

x	y	b_{in}	b_{out}	d
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

5.

id	b2	b1	b0
0	0	0	0
1	0	0	1
2	0	1	1
3	0	1	0
4	1	1	0
5	1	1	1
6	1	0	1
7	1	0	0

6. Signed magnitude & one's complement: $-127 \sim 127$
   Two's complement: $-128 \sim 127$

7. $(01000010)_2$

8. $x = 12 = 1100_2$
$y = 14 = 1110_2 = 001110_2$
By 6-bit one's complement, $x - y = x + (-y) = 001100_2 + 110001_2 = 111101_2$

9. Define eight's complement so that $-x$ is represented as n-digit $8^n - x$.
In 5-digit system, $-y = 8^5 - 15_8 = 77763_8$,
$x - y = x + (-y) = 00011_8 + 77763_8 = 77774_8$ (which equals -4)

10. (a) Proof without induction:

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \cdots + \frac{1}{(2n-1)(2n+1)}$$
$$= \frac{1}{2}(\frac{1}{1} - \frac{1}{3}) + \frac{1}{2}(\frac{1}{3} - \frac{1}{5}) + \cdots + \frac{1}{2}(\frac{1}{2n-1} - \frac{1}{2n+1})$$
$$= \frac{1}{2}(\frac{1}{1}) + \frac{1}{2}(-\frac{1}{3} + \frac{1}{3}) + \cdots + \frac{1}{2}(-\frac{1}{2n-1} + \frac{1}{2n-1}) + \frac{1}{2}(-\frac{1}{2n+1})$$
$$= \frac{1}{2}(1 - \frac{1}{2n+1})$$
$$= \frac{n}{2n+1}$$

(b) Proof with induction:
When $n = 1$, $\frac{1}{1\times 3} = \frac{1}{2+1}$, the equation is true.
Assume for $n = k$, $\frac{1}{1\times 3} + \frac{1}{3\times 5} + \cdots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$ is true,
so when $n = k + 1$, using our assumption on $n = k$,

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)}$$
$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$
$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$
$$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$
$$= \frac{k+1}{2k+3}$$

the equation is also true. Induction complete.

11. Proof: Let $x = q_x d + r_x$, $y = q_y d + r_y$,
$(x + y)\%d = (q_x d + r_x + q_y d + r_y)\%d = (r_x + r_y)\%d = (x\%d + y\%d)\%d$

12. Proof: Let $x = q_x d + r_x$, $y = q_y d + r_y$,
$(x - y)\%d = [(q_x d + r_x) - (q_y d + r_y)]\%d = (r_x - r_y)\%d = (x\%d - y\%d)\%d$

13. Proof: Let $x = q_x d + r_x$, $y = q_y d + r_y$,

$$(x \times y)\%d$$
$$= [(q_x d + r_x) \times (q_y d + r_y)]\%d$$
$$= (q_x q_y d^2 + q_x r_y d + q_y r_x d + r_x r_y)\%d$$
$$= (r_x r_y)\%d$$
$$= (x\%d \times y\%d)\%d$$