CSE20

Lecture 2: Number Systems: Binary Numbers, Gray Code, and Negative Numbers

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Number Systems

1. Introduction
2. Binary Numbers
3. Gray code
4. Negative Numbers
5. Residual Numbers

2. Binary Numbers

b_2	b_1	b_0	Value
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

This is a non-redundant number system

Examples:

3 + 5 = 8

	8	*4*	*2*	*1*	
	0	0	1	1	(3)
+	0	1	0	1	(5)
	1	0	0	0	(8)

3 + 6 = 9

	8	*4*	*2*	*1*	
	0	0	1	1	(3)
+	0	1	1	0	(6)
	1	0	0	1	(9)

2. Binary Cont.

a	b	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

RULE:

2 x Carry + Sum = a + b + c

id	a	b	c	Carry	Sum
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

2*0 + 0	=	0	0	0	id 0
2*0 + 1	=	0	0	1	id 1
2*1 + 0	=	1	1	0	id 6
2*1 + 1	=	1	1	1	id 7

3. Gray Code

id	b3	b2	b1	b0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	0	1	0
4	0	1	1	0
5	0	1	1	1
6	0	1	0	1
7	0	1	0	0
8	1	1	0	0
9	1	1	0	1
10	1	1	1	1
11	1	1	1	0
12	1	0	1	0
13	1	0	1	1
14	1	0	0	1
15	1	0	0	0

reflection

Low power (reliability) when the numbers are consecutive in series.

The idea is to only change ONE bit at a time.

e.g. addresses, analog signals

NOTE: Not for arithmetic operations (the rule is too complicated)

4. Negative Numbers

Given a positive integer x, represent the negative integer –x in (b_{n-1}, …, b_0)

(i) Signed bit system

b_{n-1}=1: negative, (b_{n-2},…,b_0)=x.

(ii) One's Complement

Present 2^n - 1 - x in binary.

(iii) Two's Complement

Present 2^n – x in binary. Ignore bit b_n.

4. Negative Numbers

NOTE: Back to binary system

(i) Signed bit - x
b3: negative

(ii) One's Complement
$2^n - 1 - x$

(iii) Two's Complement
$2^n - x$

n is the number of bits (in this case n=4)

Use the above formulas to solve for x when number is negative

One's Complement

8 = 16 - 1 - x

9 = 16 - 1 – x

Two's Complement

8 = 16 - x

Deriving One's and Two's

id	b_3	b_2	b_1	b_0	Signed	One's	Two's
0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1
2	0	0	1	0	2	2	2
3	0	0	1	1	3	3	3
4	0	1	0	0	4	4	4
5	0	1	0	1	5	5	5
6	0	1	1	0	6	6	6
7	0	1	1	1	7	7	7
8	1	0	0	0	-0	-7	-8
9	1	0	0	1	-1	-6	-7
10	1	0	1	0	-2	-5	-6
11	1	0	1	1	-3	-4	-5
12	1	1	0	0	-4	-3	-4
13	1	1	0	1	-5	-2	-3
14	1	1	1	0	-6	-1	-2
15	1	1	1	1	-7	-0	-1

Reverse Derivation

Two's	(b_4)	b_3	b_2	b_1	b_0
7	*1*	0	1	1	1
6	*1*	0	1	1	0
5	*1*	0	1	0	1
4	*1*	0	1	0	0
3	*1*	0	0	1	1
2	*1*	0	0	1	0
1	*1*	0	0	0	1
0	*1*	0	0	0	0
-1	*0*	1	1	1	1
-2	*0*	1	1	1	0
-3	*0*	1	1	0	1
-4	*0*	1	1	0	0
-5	*0*	1	0	1	1
-6	*0*	1	0	1	0
-7	*0*	1	0	0	1
-8	*0*	1	0	0	0