CSE 20 Lecture 9
Boolean Algebra

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Theorems & Proofs

- P1: $a+b = b+a$, $ab=ba$
- P2: $a+bc = (a+b)(a+c)$

  $a(b+c) = ab + ac$
- P3: $a + 0 = a$, $a1 = a$
- P4: $a + a' = 1$, $a a' = 0$

Theorem 6: for every a in B, $(a')' = a$

Proof: A is complement of a'.

The complement of a' is unique

Thus $a = (a')'$

Theorem 7: (Absorption Law) For every pair a,b in B,

$$a\cdot(a+b) = a, \quad a + a\cdot b = a$$

Proof:

$$\begin{aligned} & a(a+b) \\ &= (a+0)(a+b) && \text{(P3)} \\ &= a+0\cdot b && \text{(P2)} \\ &= a + 0 && \text{(P3)} \\ &= a && \text{(P3)} \end{aligned}$$

$$\begin{aligned} & a+a\cdot b \\ &= a\cdot 1 + a\cdot b && \text{(P3)} \\ &= a(1+b) && \text{(P2)} \\ &= a\cdot 1 && \text{(P3)} \\ &= a && \text{(P3)} \end{aligned}$$

Theorem 8

- For every pair a, b in B

  $a + a'*b = a + b, a*(a' + b) = a*b$

  Proof: $a + a'*b$

  $= (a + a')(a + b)$ by P2

  $= (1)(a + b)$ by P4

  $= (a + b)$ by P3

  $a*(a' + b)$

  $= a*a' + a*b$ (by P3)

  $= 0 + a*b$ (by P4)

  $= a*b$ (by P3)

Theorem 9: De Morgan's Law

- Theorem: For every pair a, b in set B:

  $(a+b)' = a'b'$, and $(ab)' = a'+b'$.

- Proof: We show that a+b and a'b' are complementary. According to P4, both of the following have to be true:

  $(a+b) + a'b' = 1$, $(a+b)(a'b') = 0$

Theorem 9: De Morgan's Law (cont.)

statement	justification
(a+b)+a'b' = 1	given
(a+b+a')(a+b+b') = 1	P2
(1+b)·(a+1) = 1	P4
1 · 1 = 1	P3
1 = 1	P3

statement	justification
(a+b)(a'b') = 0	given
a'b'a + a'b'b = 0	P2
0 · b' + a · 0 = 0	P1
0 + 0 = 0	P4
0 = 0	P3

5. Switching Algebra vs. Multiple Valued Boolean Algebra

- Boolean Algebra is termed Switching Algebra when B = {0, 1}
- When |B| > 2, the system is multiple valued.
  - Example:
    - M = {(0, 1, 2, 3), #, &}

#	0	1	2	3
0	0	1	2	3
1	1	1	3	3
2	2	3	2	3
3	3	3	3	3

&	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

- P1: Commutative
  - a # b = b # a
  - a & b = b & a
- P2
  - a # (b & c) = (a # b) & (a # c)
  - a & (b # c) = (a & b) # (a & c)
- P3
  - a # 0 = a
  - a & 3 = a
- P4
  - a # a’ = 3
  - a & a’ = 0

6. Boolean Transformation

Show a'b'+ab+a'b = a'+b

Proof 1:	a'b'+ab+a'b	
	= a'b'+(a+a')b	p2
	= a'b' + b	p4
	= a' + b	Theorem 8
Proof 2:	a'b'+ab+a'b	
	= a'b'+ab+a'b+a'b	Theorem 5
	= a'b' + a'b +ab+a'b	p1
	= a'(b"+b) + (a+a'b)b	p2
	= a'*1 +a*b	p4
	= a' + b	p3