CSE 20: Lecture 8
Boolean Postulates and Theorems

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Outline

- Section 1: Interpretation of Boolean Algebra using Logic Operations
- Section 2: Boolean Algebra and Gates
- Section 3: Theorems and Proofs

Section 1: Interpretation of Boolean Algebra using Logic Operations

Logic tables:

a = 1 => a is true

a OR b

id	**a b**	***a OR b***
0	0 0	0
1	0 1	1
2	1 0	1
3	1 1	1

a = 0 => a is false

a AND b

Id	**a b**	***a AND b***
0	0 0	0
1	0 1	0
2	1 0	0
3	1 1	1

P1 and P2

P1: Commutative

- a OR b = b OR a
- a AND b = b AND a

P2: Distributive

- a is true OR (b is true AND c is true)
  = (a is true OR b is true) AND (a is true OR c is true)
- a is true AND (b is true OR c is true)
  = (a is true AND b is true) OR (a is true AND c is true)

P3 and P4

P3: Identity

- 0: false, 1: true
- a is true OR false statement = a is true
- a is true AND true statement = a is true

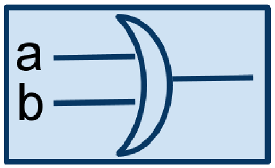
P4: Complement

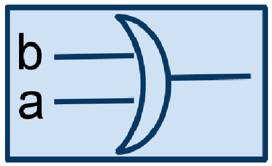
- a is true OR a is false = true
- a is true AND a is false = false

Section 2: Boolean Algebra and Gates

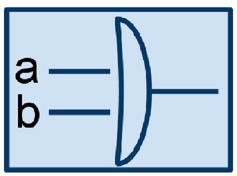
P1: Commutative

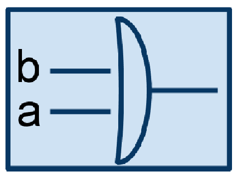
$a+b = b+a$





$a \cdot b = b \cdot a$



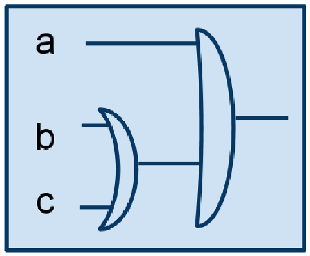


P2: Distributive

- a·(b+c) = (a·b)+(a·c)
- a+(b·c) = (a+b)·(a+c)

ID	a	b	c	b+c	a· (b+c)	~	a· b	a· c	(a· b)+(a· c)
0	0	0	0	0	0	~	0	0	0
1	0	0	1	1	0	~	0	0	0
2	0	1	0	1	0	~	0	0	0
3	0	1	1	1	0	~	0	0	0
4	1	0	0	0	0	~	0	0	0
5	1	0	1	1	1	~	0	1	1
6	1	1	0	1	1	~	1	0	1
7	1	1	1	1	1	~	1	1	1

P2: Distributive, cont.



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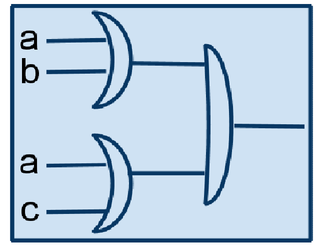
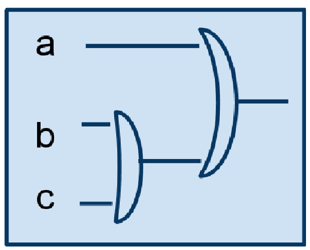


$a\cdot (b+c)$ $(a\cdot b)+(a\cdot c)$



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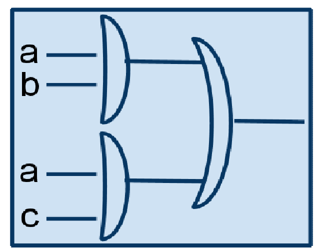
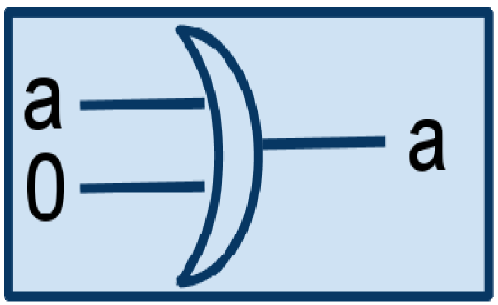


$a+(b\cdot c)$ $(a+b)\cdot (a+c)$

P3 Identity

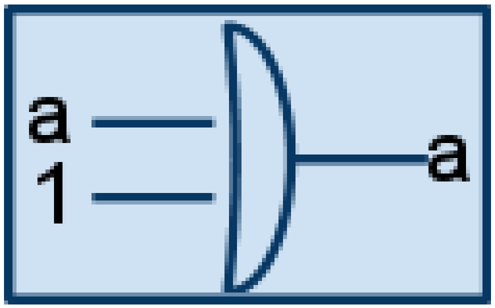
a+0 = a,

0 input to OR is passive



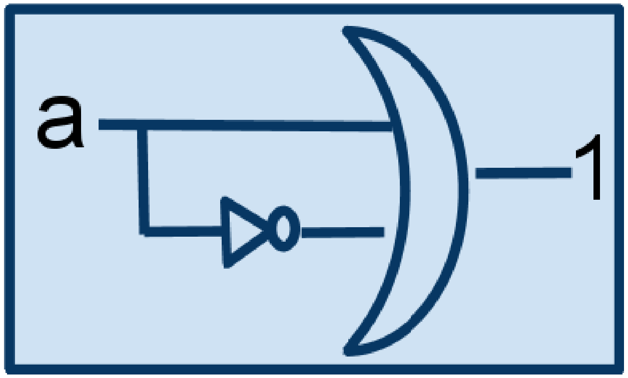
a·1 = a,

1 input to AND is passive

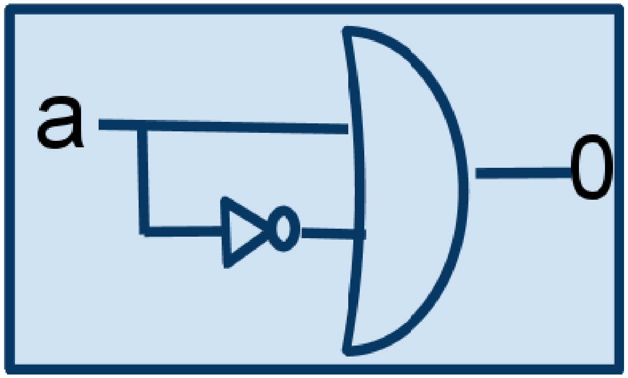


P4 Complement

$a+a' = 1$



$a \cdot a' = 0$



Section 3, Theorems and Proofs

Theorem 1: Principle of Duality

- Every algebraic identity that can be proven by Boolean algebra laws, remains valid if we swap all

  ‘+’ and ‘·’, 0 and 1

Proof:

- Visible by inspection – all laws remain valid if we interchange all

  ‘+’ and ‘·’, 0 and 1

Theorem 2

Statement: For every a Є B, its complement a' is unique.

Proof: We prove by contradiction

Suppose that a’ is not unique, i.e.

a_1' , a_2' Є B & a_1' =/= a_2’

=> $a_1' = a_1' * 1$ (Postulate 3)

=> $= a_1' * (a + a_2')$ (Postulate 4)

=> $= (a_1' * a) + (a_1' * a_2')$ (Postulate 2)

=> $= 0 + (a_1' * a_2')$ (Postulate 4)

=> $= a_1'*a_2'$

$a_1' = a_1'*a_2'$. We can also prove the same with a_2', using the same postulates, that $a_2' = a_1'*a_2'$. Therefore, we have $a_1' = a_2'$, which contradicts to our initial assumption that

a_1' =/= a_2'.

Theorem 3

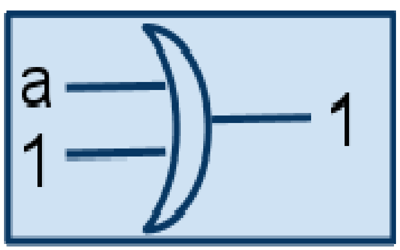
Statement: For all a Є B, a + 1 = 1, and a*0 = 0.

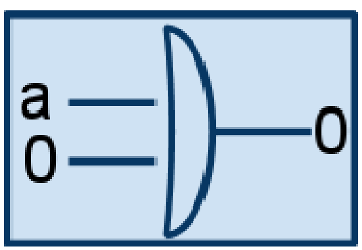
Proof: a + 1 = 1 * (a + 1) (Postulate 3)
= (a + a') * (a + 1) (Postulate 4) (Take out common property of 'a' for the next line)
= (a + a') * (1) (Postulate 2)
= a + a' (Postulate 3)
= 1 (Postulate 4)

For all a Є B,

'1' dominates as input in OR gates.

’0' dominates as input in AND gates.





Theorem 4

Statement:

- The complement of element 1 is 0 and vice versa.

$$0' = 1, 1' = 0$$

Proof:

$$0 + 1 = 1 \Rightarrow 0' = 1, 1' = 0 \text{ (Postulate 4)}$$

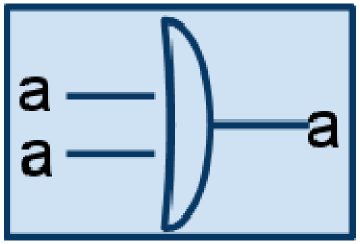
$$0 * 1 = 0 \text{ (Identity Element, Postulate 3)}$$

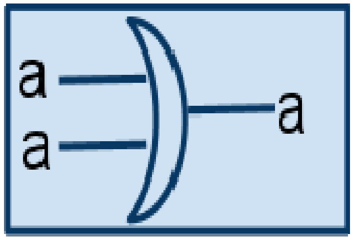
Theorem 5: Idempotent Law

Statement:

- For every a Є B,

a + a = a and a * a = a.





Proof:

a + a = (a + a) * 1 (Postulate 3)
= (a + a) * (a + a') (Postulate 4)
= a + (a*a') (Postulate 2)
= a + 0 (Postulate 4)
= a (Postulate 3)