

CSE 20: Lecture 7
Boolean Algebra
CK Cheng
2/4/2010

Outlines

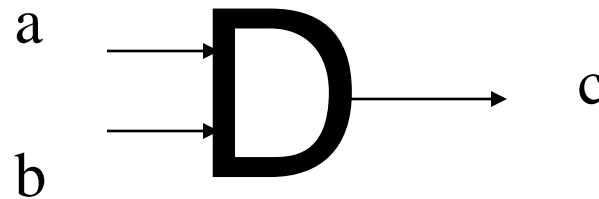
- Introduction
- Definitions
- Interpretation in Set Operations
- Interpretation in Logic Operations
- Theorems and Proofs
- Multi-valued Boolean Algebra
- Expression Transformations

1. Introduction

- Boolean algebra is used in computers for arithmetic & logic operations.
- Eg: if $a = 1$, then $y = b$, else $y = c$.
- a and b values that pass through AND gates return true if and only if both a and b are true.

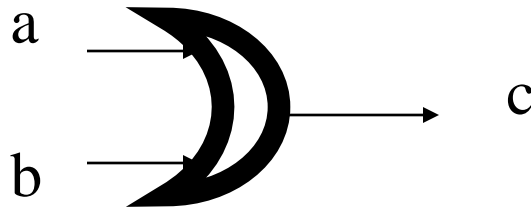
AND Gate

id	a	b	c
0	0	0	0
1	0	1	0
2	1	0	0
3	1	1	1



OR & Inverter

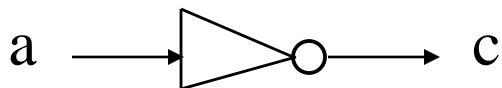
- a and b feed into OR gates. The output is true if either a or b is true.



OR Gate

id	a	b	c
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

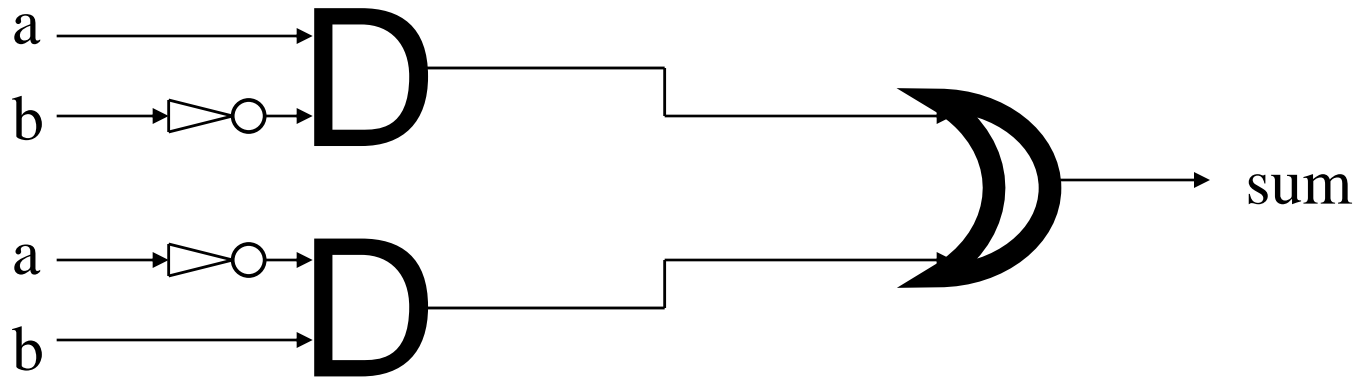
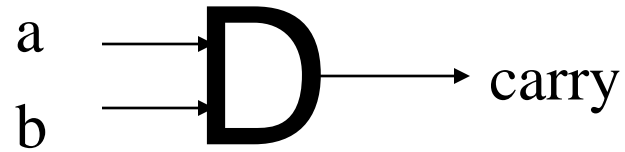
- Output c is the complement of a.



Inverter

a	c
0	1
1	0

A Half Adder:



2. Definitions

- Boolean Algebra: A set of elements B with two operations
 - $+$ (\vee , \cup , OR)
 - $*$ (\wedge , \cap , AND)

satisfying the following 4 laws:

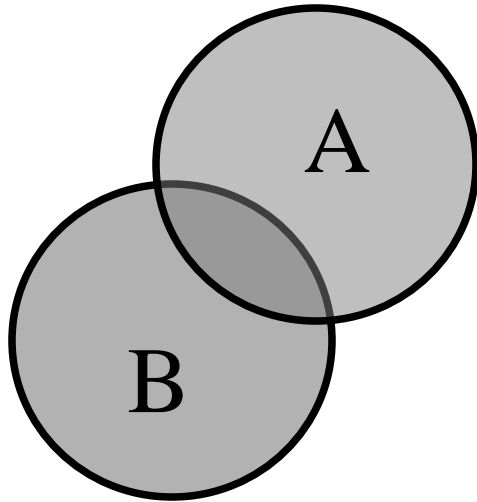
- P1: Commutative Laws: $a+b = b+a$, $a*b = b*a$.
- P2: Distributive Laws:
 $a+(b*c) = (a+b)*(a+c)$, $a*(b+c) = (a*b)+(a*c)$.
- P3: Identity Elements: Set B has two distinct elements denoted as 0 and 1 such that $a+0 = a$, $a*1 = a$.
- P4: Complement Laws: $a+a' = 1$, $a*a' = 0$.

3. Interpretation in Set Operations

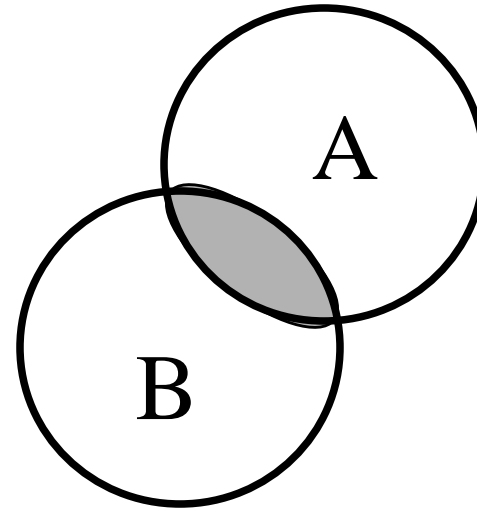
- Set: Collection of Objects
- Example:
 - $A = \{1, 3, 5, 7, 9\}$
 - $N = \{x \mid x \text{ is a positive integer}\}$, e.g. $\{1, 2, 3, \dots\}$
 - $Z = \{x \mid x \text{ is an integer}\}$, e.g. $\{-1, 0, 4\}$
 - $Q = \{x \mid x \text{ is a rational number}\}$, e.g. $\{-0.75, \frac{2}{3}, 100\}$
 - $R = \{x \mid x \text{ is a real number}\}$, e.g. $\{\pi, 12, -\frac{1}{3}\}$
 - $C = \{x \mid x \text{ is a complex number}\}$, e.g. $\{2 + 7i\}$
 - $\emptyset = \{\}$ or empty set

P1: Commutative Laws in Venn Diagram

$A+B$

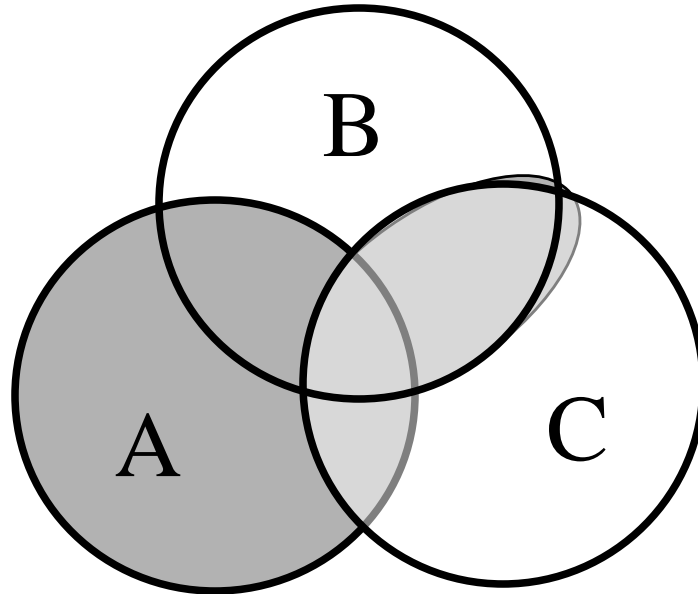


$A*B$



P2: Distributive Laws

$$A + (B * C) = (A+B)*(A+C)$$



P2: Distributive Laws

$$A * (B + C) = (A*B)+(A*C)$$

Exercise: Show the Venn Diagram

P3: Identity Elements

$$0 = \{\}$$

1 = Universe
of the set

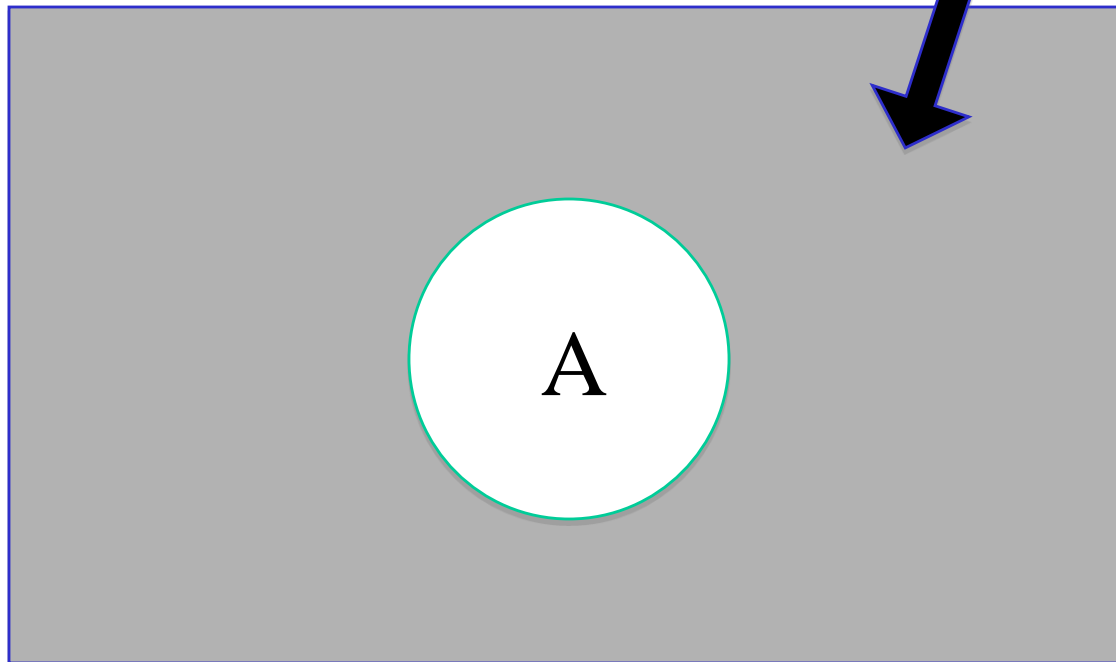
$$A+0 = A$$

$$A*1 = A$$

P4: Complement

$$A + A' = 1$$

$$A * A' = 0$$



4. Interpretation in Logic

- $A = 1 \Rightarrow A$ is true
- $A = 0 \Rightarrow A$ is false
- $A \text{ OR } B \Rightarrow$ This statement is true if A is true or B is true
- $A \text{ AND } B \Rightarrow$ This statement is true if A is true and B is true