CSE20 Lecture 5
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Solutions to Exercise 1

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Outlines

1. Hybrid System
2. Subtracter
3. Proof by Induction
4. Modulus Operations

1. Hybrid System

Problem: A hybrid system has 5 digits of radix 3 and 4 digits of radix 4. Describe the range of the system.

A	B	C	D	E
0				
1				
2				

3^5 Possibilities

A	B	C	D
0			
1			
2			
3			

4^4 Possibilities

Therefore range is from 0 to (3^5 * $4^{4)}$

2. Subtracter

Problem: Write the truth table of a full - subtracter with three binary inputs x, y, bin, and to binary outputs b, d

	X	Y	B_{in}	D	b_{out}
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	1	1
3	0	1	1	0	0
4	1	0	0	1	1
5	1	0	1	0	0
6	1	1	0	0	0
7	1	1	1	1	1

$-2\ b_{out} + d = x - y - b_{in}$

Ex.Row7

11111 //values of row7

$-2+1=1-1-1-1=-1$ //plug into equation

$-1 = -1$ //Huzzah! It 's true

Note: There is no difference between column “d” and the sum from adder

Difference:

Id	x	y	b_{in}	b_{out}	d
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	1	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	0	0
6	1	1	0	0	0
7	1	1	1	1	1

Sum:

Id	x	y	c_{in}	c_{out}	s
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

Notice:
The difference column and the sum column are the same.

3. Proof By Induction

Three Steps:

1. **Induction**

   when n = 1, the statement is true

   // solve the equation for n =1 and show it's true

2. **Assumption**

   When n = k, assume that the statement is true

   // replace the "n"s with "k"s

3. **Extension**

   When n = k+1, show that the statement is true

   // add in the k+1 at the end and try to make it look like the other equation

Basicaly:

Original equation = some outrageous claim!
Original + (original with k+1) = claim + (oritinal with k+1)
Make the "claim + (original with k+1)" look like "claim with k+1"

10. Want to show

$$\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$$

For $n = 1$, $\frac{1}{(1)(3)} = \frac{1}{2(1)+1}$ true. Assuming true for n,

$$\begin{aligned}
\sum_{i=1}^{n+1} \frac{1}{i(i+2)} &= \sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} + \frac{1}{(2n+1)(2n+3)} \\
&= \frac{n}{2n+1} + \frac{1}{(2n+1)(2n+3)} \\
&= \frac{n(2n+3)}{(2n+1)(2n+3)} + \frac{1}{(2n+1)(2n+3)} \\
&= \frac{2n^2+3n+1}{(2n+1)(2n+3)} \\
&= \frac{(n+1)(2n+1)}{(2n+1)(2n+3)} \\
&= \frac{n+1}{2(n+1)+1}
\end{aligned}$$

Therefore, the supposition is true $\forall n \in \mathbb{N}$.

NB. To develop the right-hand side of the equation in the supposition, Actually perform the sums and try to fit a pattern to it. If you think you have the right answer, try to prove it by induction, as above.

4. Modulus Operations

- set x/y = $q_{(x/y)}$ * n + remainder
- plug into the left side of the equation
- try to make it look like the other side of the equation

11.Given three integers x,y,d, prove that

(x+y)%d=(x%d+y%d)%d, where % is a modulus operation

Let x = q_xd + r_x, y = q_yd + r_y //start with this

(x+y)%d = (q_xd + r_x + q_yd + r_y)%d //plug in x & y

(qxd)%d + (r_x)%d + (q_yd)%d + (r_y)%d //distribute the %

0+(r_x)%d + 0 + (r_y)%d //d%d=0

r_x%d + r_y%d = (r_x + r_y)%d //factor out %d

If you remember, r_x is remainder of x/d (or r_x= x%d)

(r_x+r_y)%d = (x%d + y%d)%d //rx=x%d&ry=y%d

Therefore,(x+y)%d = (x%d+y%d)%d //YAY! We win :3

11. *Show* $(x+y)\%d = (x\%d + y\%d)\%d$.

For $x = a_1 d + a_2$, $y = b_1 d + b_2$,

$$\begin{aligned}(x+y)\%d &= (a_1 d + a_2 + b_1 d + b_2)\%d \\ &= a_1 d\%d + a_2\%d + b_1 d\%d + b_2\%d \\ &= 0 + a_2\%d + 0 + b_2\%d \\ &= (a_2 + b_2)\%d\end{aligned}$$

Since $a_2 = x\%d$ and $b_2 = y\%d$, $(x+y)\%d = (x\%d + y\%d)\%d$.

12. *Show* $(x-y)\%d = (x\%d - y\%d)\%d$.

For $x = a_1 d + a_2$, $y = b_1 d + b_2$,

$$\begin{aligned}(x-y)\%d &= (a_1 d + a_2 - b_1 d - b_2)\%d \\ &= a_1 d\%d + a_2\%d - b_1 d\%d - b_2\%d \\ &= 0 + a_2\%d - 0 - b_2\%d \\ &= (a_2 - b_2)\%d\end{aligned}$$

Since $a_2 = x\%d$ and $b_2 = y\%d$, $(x-y)\%d = (x\%d - y\%d)\%d$.

13. *Show* $(xy)\%d = (x\%d)(y\%d)\%d$.

For $x = a_1 d + a_2$, $y = b_1 d + b_2$,

$$\begin{aligned}(xy)\%d &= [(a_1 d + a_2)(b_1 d + b_2)]\,\%d \\ &= \left[a_1 b_1 d^2 + (a_1 b_2 + a_2 b_1)d + a_2 b_2\right]\%d \\ &= (a_1 b_1 d^2)\%d + ((a_1 b_2 + a_2 b_1)d)\%d + (a_2 b_2)\%d \\ &= 0 + 0 + (a_2 b_2)\%d\end{aligned}$$

Since $a_2 = x\%d$ and $b_2 = y\%d$, $(xy)\%d = (x\%d)(y\%d)\%d$.

NB. why not also prove $(x/y)\%d \stackrel{?}{=} ((x\%d)/(y\%d))\%d$*? Because taking* x/y *when* $x \neq \alpha y$ *is undefined (modulus is only defined for integers). Also, when* $x = \alpha y$ *and* $y\%d = 0$*, the division is undefined. Therefore, The supposition is not always true.*