CSE 20 Lecture 4

01/14/10

CK Cheng, UC San Diego

- Negative numbers

  One's and Two's Complement

- Residual numbers

Announcements

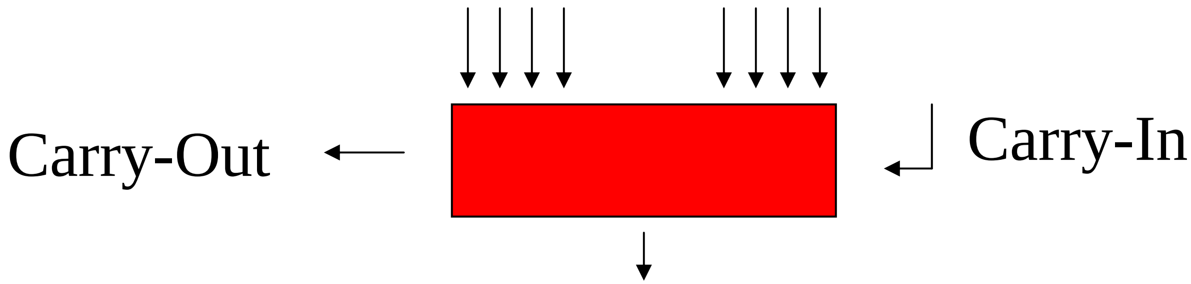
- CK No office hours next week
- Pat No office hours next week
- On Tuesday 1/19, Peng Du will be going over Chapter 11(Shaum's)
- On Thursday 1/21, Peng Du will be going over Chapter 11 (Shaum's)
- Midterm will be held on 1/28

One's Complement

- Given two positive integers x & y, perform:
  - $x + y$
  - $x - y$
  - $-x + y$
  - $-x - y$

How to Solve

1) Derive 1’s complement of the operands
2) Sum up the two operands
3) Use carry-out to feed into carry-in
4) The result is the solution in 1’s complement



Formulas for 1's Complement

Arithmetic	1's Complement
$x+y$	$x+y$
$x-y$	$x+(2^n-1-y)=2^n-1+(x-y)$
$-x+y$	$(2^n-1-x)+y = 2^n-1+(-x+y)$
$-x-y$	$(2^n-1-x)+(2^n-1-y)=2^n-1+(2^n-1-x-y)$

1's Complement

Example: 4 – 3 = 1

3 in binary = 0011. Flipping the bits, you get –3 (1100) in binary, which is 12.

0100	(4 in binary)
+1100	(12 in binary)
10000	=16 (15+1)

So now take the extra 1 and remove it from the 5th spot and add it to the remainder

0000	
+ 1	
0001	(1 in 1's comp)

1's Complement

Example: – 4 + 3 = – 1

4 in binary = 0100. Flipping the bits, you get –4 (1011) in binary, which is 11.

1011	(11 in binary)
+0011	(3 in binary)
1110	(=14 in binary (15-1), but in 1's comp, is -1)

1's Complement

Example: – 4 – 3 = – 7

4 in binary = 0100. Flipping the bits, you get –4 (1011) in binary, which =11.

3 in binary = 0011. Flipping the bits, you get –3(1100) in binary, which =12.

1011	(11 in binary, or 15-4)
+1100	(12 in binary, or 15-3)
10111	(=23 in binary (15+15-7))

So now take the extra 1 and remove it from the 5th spot and add it to the remainder

0111	
+ 1	
1000	(-7 in 1's comp)

Recovery of the Numbers

1's Compliment

Let $f(x) = 2^n - 1 - x$

Theorem: $f(f(x)) = x$

Proof: $f(f(x))$

$= f(2^n - 1 - x)$

$= 2^n - 1 - (2^n - 1 - x)$

$= x$

2's Compliment

Let $g(x) = 2^n - x$

Theorem: $g(g(x)) = x$

Proof: $g(g(x))$

$= g(2^n - x)$

$= 2^n - (2^n - x)$

$= x$

Residual Numbers
(NT-1, Chp 11 Shaums)

1) Introduction

2) Definitions

3) Operations

Residual Numbers - Introduction

Goal: Simplify arithmetics (+ – x) when bit width n is huge, eg. n=1000.

Definition: Mod (*Modulus*) operation

$$\text{Integer } x = q * d + r, \quad 0 <= r < d,$$

where q : quotient, d : divisor, and r : remainder.

Mod: x%d =r

Examples:

0%3 =0	2%3 =2	21%3 =0
0%5 =0	2%5 =2	21%5= 1
0%7 =0	2%7 =2	21%7= 0

Negative Numbers:

-3%3 =0

-3%5 =2   to solve: -3= q*5+r

q has to be equal to –1 as the remainder can only be positive

-3%7 =4   to solve: -3 = q*7+r

q has to be equal to –1 as the remainder can only be positive

-21%3 =0

21%5 = 4   to solve: -21=q*5+r

q=-5, r =4 as -21= -25+4