CSE20 Lecture 2W1

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Number Systems: Negative Numbers

1. Sign and Magnitude Representation
2. 1's Complement Representation
3. 2's Complement Representation

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Outlines

1. Goal of the Negative Number Systems
2. Definition
    1. Sign Magnitude Rep.
    2. 1's Complement Rep.
    3. 2.s Complement Rep.
3. Arithmetic Operations

Goal of negative number systems

- Signed system: Simple. Just flip the sign bit
    - 0 = positive
    - 1 = negative
- One's complement: Replace subtraction with addition
    - Easy to derive (Just flip every bit)
- Two's complement: Replace subtraction with addition
    - Addition of one's complement and one produces the two's complement.

Definitions: Given a positive integer x, we represent -x

- 1's complement:
  - Formula: $2^n - 1 - x$
    - i.e. n=4, $2^4 - 1 - x = 15 - x$
    - In binary: $(1\ 1\ 1\ 1) - (b_3\ b_2\ b_1\ b_0)$
    - Just flip all the bits.
- 2's complement:
  - Formula: $2^n - x$
    - i.e. n=4, $2^4 - x = 16 - x$
    - In binary: $(1\ 0\ 0\ 0\ 0) - (0\ \ b_3\ b_2\ b_1\ b_0)$
    - Just flip all the bits and add 1.

Definitions: 4-Bit Example

id	b_3	b_2	b_1	b_0	Signed	One's	Two's
0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1
2	0	0	1	0	2	2	2
3	0	0	1	1	3	3	3
4	0	1	0	0	4	4	4
5	0	1	0	1	5	5	5
6	0	1	1	0	6	6	6
7	0	1	1	1	7	7	7
8	1	0	0	0	-0	-7	-8
9	1	0	0	1	-1	-6	-7
10	1	0	1	0	-2	-5	-6
11	1	0	1	1	-3	-4	-5
12	1	1	0	0	-4	-3	-4
13	1	1	0	1	-5	-2	-3
14	1	1	1	0	-6	-1	-2
15	1	1	1	1	-7	-0	-1

Definitions: Examples

Given n-bits, what is the range of my numbers in each system?

- 3 bits:
  - Signed: -3 , 3
  - 1's: -3 , 3
  - 2's: -4 , 3
- 6 bits
  - Signed: -31, 31
  - 1's: -31, 31
  - 2's: -32, 31
- 4 bits:
  - Signed: -15, 15
  - 1's: -15, 15
  - 2's: -16, 15
- Given 8 bits
  - Signed: -127, 127
  - 1's: -127, 127
  - 2's: -128, 127

Formula for calculating the range →

Signed & 1's: $-(2^{n-1} - 1)$, $(2^{n-1} - 1)$

2's: -2^{n-1} , $(2^{n-1} - 1)$

Arithmetic Operations:
Derivation of 1's Complement

Theorem 1: For 1's complement, given a positive number (x_{n-1} , x_{n-2} , … , x_0), the negative number is

$(\bar{x}_{n-1}, \bar{x}_{n-2}, \ldots, \bar{x}_0)$ where $\bar{x} = 1 - x$

Proof:

(i). 2^n-1 in binary is an n bit vector (1,1, …, 1)

(ii). 2n-1-x in binary is (1,1, …,1) – (x_{n-1},x_{n-2}, …, x_0). The result is

$$(\bar{x}_{n-1}, \bar{x}_{n-2}, \ldots, \bar{x}_0)$$

Arithmetic Operations: Derivation of 2's Complement

Theorem 2: For 2's complement, given a positive integer x, the negative number is the sum of its 1's complement and 1.

Proof: $2^n - x = 2^n - 1 - x + 1$. Thus, the 2's complement is

$$(\bar{x}_{n-1}, \bar{x}_{n-2}, \ldots, \bar{x}_0) + (0, 0, \ldots, 1)$$

Ex: x = 9 (01001)

1's -9 (10110)

31 – 9 = 22

2's -9 (10111)

32 – 9 = 23

Ex: x = 13 (01101)

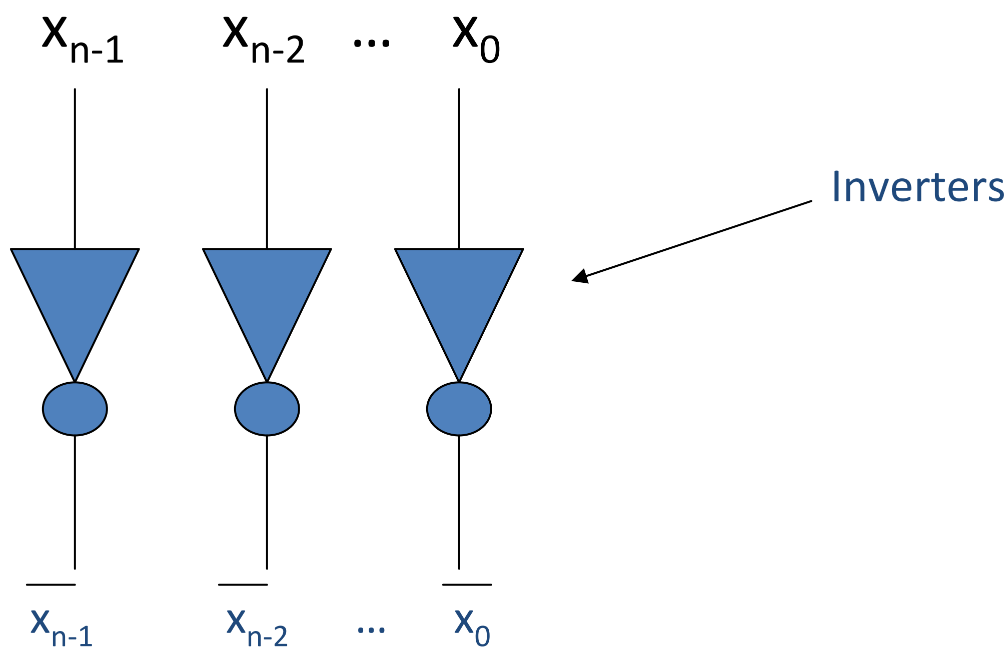
1's -13 (10010)

31-13=18

2's -13 (10011)

32-13=19

One's Complement Hardware:



Arithmetic Operations: 2's Complement

Input: two positive integers x & y,

1. We represent the operands in two's complement.
2. We sum up the two operands and ignore bit n.
3. The result is the solution in two's complement.

Arithmetic	2's complement
$x + y$	$x + y$
$x - y$	$x + (2^n - y) = 2^n+(x-y)$
$-x + y$	$(2^n - x) + y =2^n+(-x+y)$
$-x - y$	$(2^n - x) + (2^n - y) = 2^n+2^n-x-y$

Arithmetic Operations: Example: 4 – 3 = 1

$4_{10} = 0100_2$

$3_{10} = 0011_2$ $-3_{10} \rightarrow 1101_2$

$$
\begin{array}{r}
0100 \\
\underline{+\ 1101} \\
10001
\end{array}
\rightarrow 1 \text{ (after discarding extra bit)}
$$

We discard the extra 1 at the left which is 2^n from 2's complement of -3. Note that bit b_{n-1} is 0. Thus, the result is positive.

Arithmetic Operations: Example: -4 +3 = -1

$4_{10} = 0100_2$ -4_{10} → Using two's comp.→ $1011 + 1 = 1100_2$

(Invert bits)

$3_{10} = 0011_2$

  1100
+ 0011
  1111 → Using two's comp. → 0000 + 1 = 1, so our answer is -1

If left-most bit is 1, it means that we have a negative number.

Arithmetic Operations: Example: -4 – 3 = -7

$4_{10} = 0100_2$ -4_{10} → Using two's comp. → $1011 + 1 = 1100_2$

(Invert bits)

$3_{10} = 0011_2$ -3_{10} → Using two's comp. → $1100 + 1 = 1101_2$

(Invert bits)

  1100
+ 1101

11001 → discard bit 4 → 1001 (-7)

We discard bit 4. Bit 3 is one. Thus, it is a negative number.