CSE 20 Lecture 11
Function, Recursion & Analysis
(Ch. 6 Shaum's)
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OUTLINE

- DEFINITION
- FUNTION RECURSION: CASES
- ANALYSIS

I. Definition

- A function f: A → B maps elements in domain A to codomain B such that for each a ϵ A, f(a) is exact one element in B.
- f: A → B

  A: Domain

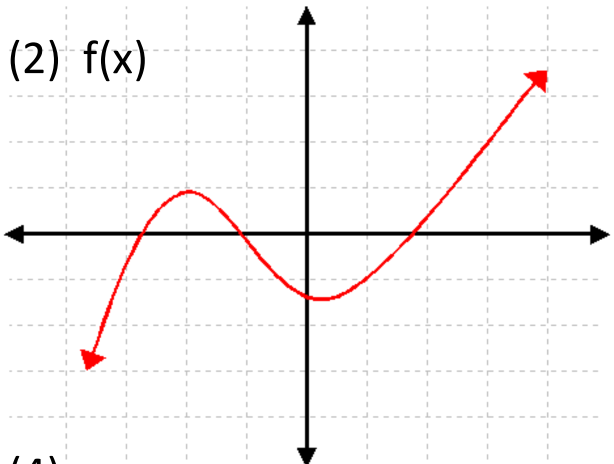
  B: Codomain

  f(A): range or image of function f(A) ⊂ B

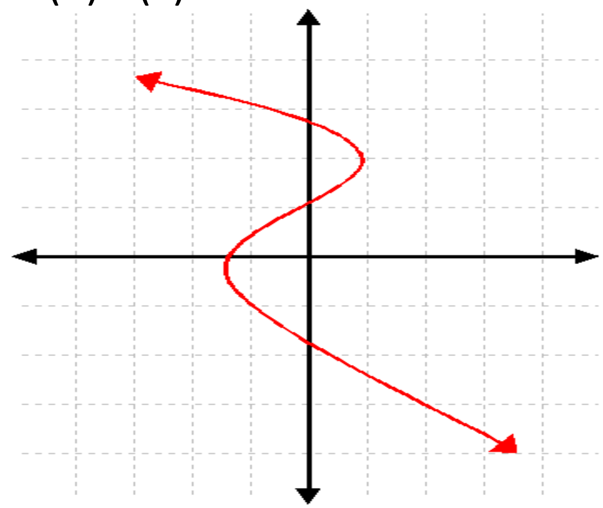
Examples

(1) $f(x)=x^2$, $x \in \mathbb{R}$

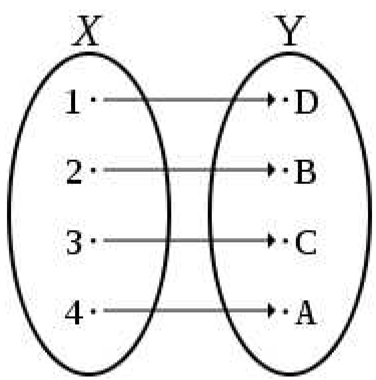
(2) f(x)



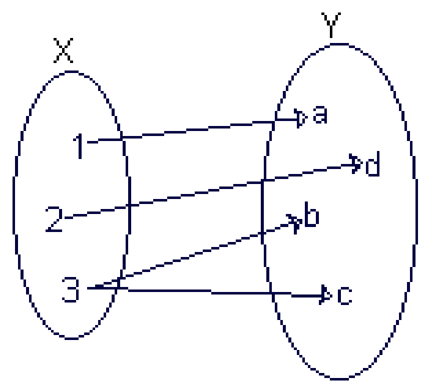
(3) f(x) NOT A FUNCTION



(4)



(5) NOT A FUNCTION



6) if domain A is an integer set Z, we may denote f(x) as f_x, ie. f_0, f_1, f_2.

Cases of Recursion

- Fibonacci Sequence

$$f_0 = 0,\ f_1 = 1,\ f_n = f_{n-1} + f_{n-2} \qquad \forall_n > 1$$

0 1 2 3 4 5 6

0 1 1 2 3 5 8

Cases of Recursion: Ackermann Function

- Ackermann Function: A(m,n), m, n $\in N_0$
- a) m=0, A(0,n)=n+1
- B) m $\neq$ 0, n=0; A(m,0)=A(m-1,1)
- C) m $\neq$ 0, n $\neq$ 0, A(m,n)=A(m-1, A(m,n-1))

- Example:

  A(1,1)= A(0, A(1,0)) =A(0,2)

  A(2,0) =A(1,1)

Ackermann Function

- (0,0) (0,1) (0,2) (0,3) (0,4) (0,5) (0,6) (0,7)
  1 2 3 4 5 6 7 8 A(0,n)=n+1
- (1,0) (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (1,7)
  2 3 4 5 6 7 8 9 A(1,n)=n+2
- (2,0) (2,1) (2,2) (2,3) (2,4)(2,5)
  3 5 7 9 11 13 A(2,n)=2n+3
- (3,0) (3,1) (3,2)
  5 13 29 A(3,n)= $2^{n+3}-3$
- (4,0)(4,1)
  13 65533

A(4,n)= $\underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}_{n+3 \text{ 2's}}$ -3

$$A(4,1)=2^{2^{2^{2}}}-3=2^{2^{4}}=2^{16}-3=65536-3$$

Ackermann Function - Comparison

- Ackermann Function: grows faster than an exponential function

  $$A(4,2) = 2 \times 10^{19728}$$

- Googol: $10^{100} \approx 70!$
- Visible universe $\approx 10^{81}$ atoms

Fibonacci Sequence

- Fibonacci sequence:

Start with a pair of rabbits. Every month each pair bears a pair, which becomes productive from the second month. Calculate the number of new pairs in month i, $i \geqslant 0$.

Analysis of Fibonacci Sequence

- Algorithm

array n(i) (new born), a(i) (adult)

Init n(0)=0, a(0)=1

For i=1, i=i+1

    n(i)=a(i-1)

    a(i)=a(i-1)+n(i-1)

Index	0	1	2	3	4	5	6	…
a(i)	1	1	2	3	5	8	13	…
n(i)	0	1	1	2	3	5	8	…

Fibonacci sequence: Golden Ratio

$$X_n = \frac{f_n}{f_{n-1}} \qquad X_{n\to\infty} = \frac{1+\sqrt{5}}{2}$$

- Derivation:

$$\frac{f_n}{f_{n-1}} = \frac{f_{n-1} + f_{n-2}}{f_{n-1}} = 1 + \frac{f_{n-2}}{f_{n-1}} = 1 + \frac{1}{\frac{f_{n-1}}{f_{n-2}}}$$

- Let $X = X_{n\to\infty}$
- We have: $X = 1 + {}^{1}\!/_{X}$

Fibonacci Sequence and the golden ratio

n	0	1	2	3	4	5	6	7	8	9
f_n	0	1	1	2	3	5	8	13	21	34
$\frac{f_n}{f_{n-1}}$			1	2	1.5	1.6	1.61	1.625	1.615	1.619