

# CSE 20

## Discrete Mathematics

### **Instructor**

CK Cheng, CSE2130, ckcheng+20@ucsd.edu, tel: 858  
534-6184

### **Teaching Assistants**

#### **Pat Rondon**

E-mail: prondon@cs.ucsd.edu

Office Hours: 1:00PM - 2:00PM M, 10:00-  
11:00AM F, CSE B240A

#### **Renshen Wang**

E-mail: rewang@cs.ucsd.edu

Office Hours: 3:00PM - 5:00PM T, 8:00 -  
10:00PM Th, CSE B240A

<http://cseweb.ucsd.edu/classes/wi10/cse20/>

# Textbooks

- [A Short Course in Discrete Mathematics](#)  
Edward A. Bender and S. Gill Williamson, Dover, 2005.
- **Discrete Mathematics**  
Seymour Lipschutz and Marc Lipson, Schaum's Outline Series,  
Third Edition, McGraw Hill, 2007

# Grading

- Quizzes and Scripts 5%
- Midterm 1 25%  
01/21/2010
- Midterm 2 30%  
02/16/2010
- Final Exam 40%  
03/16/2010 Tuesday from 7:00-10:00

# Administrative

- **Schedule**

- Lectures: 6:30-7:50PM TTh, CSB001.
- Discussion: 10:00-10:50AM W, Center 113.

- **First Discussion Section: Wednesday, Jan. 13**

- Office hours: 10:30-11:30AM T, 11:00-12:00PM Th, CSE 2130.

# Course Outline

**Part 1. Numbers:** choice of number systems, binary, Gray code, one's complement, two's complement, residual number system, theorems of primes and modulations.

**Part 2. Boolean Algebra:** manipulation of logic and gates, laws and theorems, tautology, SAT, multiple elements, minimization.

**Part 3. Functions and Recursion:** function definition and calculation, induction process, k'th order series, Factorial, Fibonacci, Ackerman, division, square root iterations.

# Part I. Number Systems

1. Introduction
2. Binary Number B.F. Section 2
3. Gray Code
4. Negative Numbers B.F. Section 2
5. Residual Numbers N.F. Section 1, Shaum Ch. 11

# I. Introduction

1. Examples of different number systems
2. Efficiency of the systems
3. Remarks

# 1. Example of Number Systems

## Radices (or Bases)

- Decimal
  - Each digit has a weight of 0-9 with each place multiplied by 9
  - Radix 10 → increase each place holder by 10
  - Example:  $250 = 2 * 10^2 + 5 * 10$
- Binary
  - Each place has a weight of 2
  - Radix 2
  - Example: 11010, can sum weights →  $2^4 + 2^3 + 2 = 26$
- Ternary
  - The weight of each place advances by a factor of 3
  - Radix 3
- Hybrid
  - Places have varying weights
  - Example: time
  - 1 year 3 months 2 days 1 hour 4 min 26 seconds



# 1. Examples (cont.)

- **Binary (radix 2)**
  - 2 symbols possible in each place (0, 1).
  - With  $n$  digits, we need  $2n$  tokens with 2 tokens per digit.
  - With  $n$  digits,  $2^n$  numbers can be represented
- **Ternary (radix 3)**
  - 3 symbols possible in each place (0, 1, 2).
  - With  $n$  digits, we need  $3n$  tokens
  - With  $n$  digits,  $3^n$  numbers can be represented
- **(radix 5)**
  - 5 symbols possible in each place (0, 1, 2, 3, 4).
  - With  $n$  digits, we need  $5n$  tokens
  - With  $n$  digits,  $5^n$  numbers can be represented
- **Decimal (radix 10)**
  - 10 symbols possible in each place (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).
  - With  $n$  digits, we need  $10n$  tokens
  - With  $n$  digits,  $10^n$  numbers can be represented

## 2. Efficiency of Number Systems

How many numbers can we represent in each system with 30 symbols?

- **Binary**
  - $30=2n \rightarrow$  The length of the number is  $n=15$
  - $2^{15} \sim 33,000$
- **Ternary**
  - $30=3n \rightarrow$  The length of the number is  $n=10$
  - $3^{10} \sim 60,000$
- **Radix 5**
  - $30=5n \rightarrow$  The length of the number is  $n=6$
  - $5^6 \sim 16,000$
- **Decimal**
  - $30=10n \rightarrow n=3$
  - $10^3 \sim 1000$

# What's Most Expressive?

- For radix  $k$ , with  $n$  digits
  - Range is  $k^n$
  - # tokens required is  $kn=t$
  - We maximize the range at a constant #token=  $t$ 
    - $k^n=k^{t/k} \rightarrow$  In real space, the solution is  $k=e$
    - $e$  is closest to the integer 3
  - As seen in previous slide, ternary number systems can represent the most numeric values for a given number of tokens.
  - However ternary is difficult to implement, so binary is used in computer systems.

# 3. Remarks

- We design number systems according to the usages and technologies.
- For VLSI designs, binary number system is consistent with the technology.
- Various number systems are possible for different goals and technologies, e.g. low power, reliability, security, bandwidth.