CSE 20 - Winter 2008
Quiz 5: 2008-02-19 SOLUTIONS

You have 10 minutes to take this quiz.

1. (*6 points*) Prove by induction that $\forall n \in \mathbb{Z}^+, 2^n < (n+2)!$.

- Basis step ($n = 0$): Let $n = 0$. Then $2^n = 2^0 = 1 < (0+2)! = 2$.
- Inductive hypothesis: Assume for $k \geq 0$, the claim holds: $2^k < (k+2)!$.
- Inductive step: We must prove the claim for $k+1$. We want to prove $2^{k+1} < (k+2+1)!$.

$$\begin{aligned}
2^{k+1} &= 2 \cdot 2^k && \text{factoring out 2} \\
&< 2(k+2)! && \text{by the induction hypothesis} \\
&< (k+3)(k+2)! && \text{since } 2 < 3 \leq 3+k,\ \forall k \geq 0 \\
&= (k+3)! && \text{by definition of factorial} \\
&= (k+1+2)!
\end{aligned}$$

$\square$

2. (*4 points*) Let $A = \{1,3,5,7,9\}$, $B = \{3,6,9\}$, and $C = \{2,3,4,6,8\}$. Find each of the following:

(a) $B - A = \{6\}$.

(b) $B \cap C = \{3,6\}$.