Name:
Student ID number:

prob.	score
1	
2	
3	
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total	

1. Suppose $f : A \to B$. Define the inverse set as

$$f^{-1}(b) = \{a \in A \mid f(a) = b\} \quad \text{for } b \in B.$$

Note that $f^{-1}(b)$ is a *set*. Prove that the collection of these inverse sets

$$\{f^{-1}(b) \mid b \in B\}$$

is a partition of A. *Hint:* You need to show two properties. First, prove that for all $a \in A$, there is some b such that $a \in f^{-1}(b)$. Second, show that each $a \in A$ belongs to *only one* set $f^{-1}(b)$ (and hence the sets $f^{-1}(b)$ must be disjoint).

2. Suppose A is countable and B is uncountable. Is $A \cap B$ countable? Is $A \cup B$ countable? Why?

3. Prove by induction that

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3} \quad \text{for } n \geq 1.$$

4. Define the *fibonacci* sequence as

$$f_1 = 1$$
$$f_2 = 1$$
$$f_i = f_{i-1} + f_{i-2} \quad \text{for } i \geq 3.$$

Using induction, prove that f_{3k} is even for all $k \geq 1$ (*e.g.* f_3 is even, f_6 is even, *etc.*).

5. Let d and k be positive integers. Define a relation R on $\mathbb{Z}$ as

$$(x, y) \in R \text{ if } d | \left(x^k - y^k\right).$$

Prove that R is an equivalence relation.

6. Prove that for any sets A, B, C

$$(A - B) \cap (A - C) = A - (B \cup C).$$

7. How many elements does $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$ have? ($\mathcal{P}(A)$ denotes the powerset of A).

8. A relation R on A is *circular* if for all $x, y, z \in A$, xRy and yRz implies zRx. Show that a reflexive circular relation is an equivalence relation.

9. Suppose that $f : A \to B$, $g : B \to C$ are both onto. Prove that $g \circ f$ is onto.