prob.	score
1	
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total	

Name:
Student ID number:

This exam is due by Saturday, 8/4 at 11am. To turn it in, pick one of the following:

- Drop it off in person between 8am and 11am on Saturday in WLH 2114 (two doors down from the regular class room).
- Drop it off in Scott Yilek's CSE mailbox (the mailroom is on the second floor of the CSE building). Note that the building is locked except during business hours.
- Email it to `lcayton@cs.ucsd.edu` and `syilek@cs.ucsd.edu`. PDF only please.

You may use your book and notes, but no collaboration is permitted. Please check the website for corrections to the exam.

1. Prove that for all sets A and B

$$(A \cup B) \cap (A \cup B^c) = A.$$

2. The following truth table defines $f(p,q,r)$.

p	q	r	$f(p,q,r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

Write a (logic) formula for $f(p,q,r)$. Show algebraically that your formula is equivalent to $(p \Rightarrow q) \wedge r$.
Hint: factor out the r first.

3. Show that gcd(a, b) divides lcm(a, b), where $a, b \in \mathbb{Z}$. *Hint*: recall the definitions. gcd(a, b) is the greatest positive integer that divides both a and b; lcm(a, b) is the least positive integer c such that $a|c$ and $b|c$.

4. Recall that relation on a set A is a subset of $A \times A$. Suppose that R_1 and R_2 and both equivalence relations on A. Is $R_1 \cap R_2$ necessarily an equivalence relation? Is $R_1 \cup R_2$ necessarily an equivalence relation? Prove your answers.

5. Prove using induction that

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1 \quad \text{for } n \geq 1.$$

Recall that $k! = k(k-1)(k-2)\cdots 1$.

6. Suppose that $\{A_1, A_2, \ldots, A_m\}$ and $\{B_1, B_2, \ldots, B_n\}$ are each partitions of a set X. Prove that

$$P = \{A_i \cap B_j \mid i = 1, \ldots, m, \;\; j = 1, \ldots, n\}$$

is also a partition of X.

7. Let $f : A \to B$ and $g : B \to C$. Prove that if $g \circ f$ is onto then g must be onto. Give an example to show that that f does *not* have to be onto when $g \circ f$ is.