

Assignment #1

(Due date: 2/1/07)

1. F&P 1.2 [2pts]

2. F&P 1.3 [3pts]

3. Thin Lens Equation [3pts]

An illuminated arrow forms a real inverted image of itself at a distance $w = 40\text{cm}$, measured along the optic axis of a convex thin lens (see figure 1). The image is just half the size of the object.

- How far from the object must the lens be placed?
- What is the focal length of the lens?
- If the index of refraction for the lens is $\eta = 1.5$, what is the lens radius?
- Suppose the arrow moves 5cm to the right while the lens and image plane remain the fixed. This will result in an out of focus image; what is the radius of the corresponding blur circle formed from the tip of the arrow on the image plane assuming the diameter of the lens is d ?

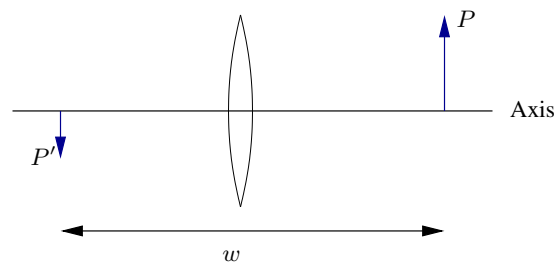


Figure 1: Problem 3. setup.

4. F&P 2.5 [2pts]

5. F&P 2.8 [3pts]

6. Affine Pose Estimation [3pts]

An affine camera transforms 3d (homogeneous) points $\mathbf{x} = (x, y, z, 1)^T$ according to,

$$\mathbf{x}' = \mathbf{M}\mathbf{x} \quad (1)$$

where

$$\mathbf{M} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

is the camera matrix and $\mathbf{x}' = (x', y', 1)^T$ is the mapping onto the image plane.

Given a set of *corresponding* points $\mathbf{x}$ and $\mathbf{x}'$, your task is to recover the affine camera parameters.

- What is the minimum number of corresponding points needed to recover the camera parameters?
- Are there any conditions on the positions of these correspondence points for successful recovery of $\mathbf{M}$? List two degenerate configurations. Can you think of a necessary condition?
- Running `house_model.m` will display an image of an affine transformed house. A set of 8 corresponding points are contained in the matlab variables `Xcorr` (original points) and `Xcorr_` (affine transformed points). Write a matlab function that takes as input `Xcorr` and `Xcorr_` and returns the affine transformation matrix, M . What is the recovered affine transformation matrix?
- Repeat part (c) using only correspondence points 1,3,4, and 6. What happens when you try to estimate M ? Why?
- Write a second program that prompts the user to manually click the correspondence points from the plot of the house model shown after running `house_model.m`¹. Using the manually selected correspondences, compute the affine transformation matrix M . Do you get the same result as in part (c)? How sensitive is the estimation to errors in the correspondences (experiment by purposely mis-clicking some of the points)?

What to turn in : Answer the questions and print out your code from parts (c) and (e).

7. Irradiance 1 [3pts]

- Consider a cylinder with radius r and height h whose base is centered at $z = 0$ along the xy -plane. If the walls of the cylinder have constant radiance L and the top of the cylinder has constant radiance $2L$, what is the irradiance E at the point $(0, 0, 0)$ assuming that the surface at $(0, 0, 0)$ has a normal vector of $(0, 0, 1)$? (See figure 2).
- What is the irradiance if the top has constant radiance L (i.e., the entire cylinder has constant radiance L)?

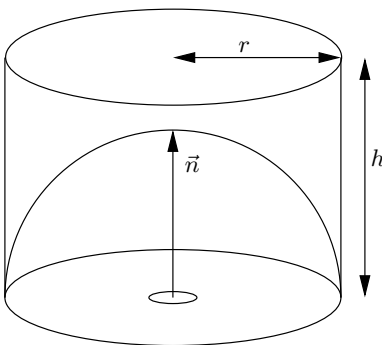


Figure 2: Problem 7. setup.

¹The matlab command 'ginput' will be useful here.

8. Irradiance 2 [6pts]

Consider a rectangular surface with vertices $(-2, -1, 1)$, $(-2, 1, 1)$, $(2, 1, 1)$, and $(2, -1, 1)$. If the radiance on the surface is $L(x^2 + y^2 + 1)$, what is the irradiance arriving at position $(0, 0, 0)$ with normal vector $(0, 0, 1)$? (See figure 3).

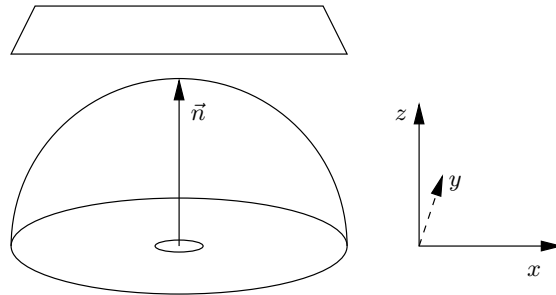


Figure 3: Problem 8. setup.