

Extra-credit recursion problems

due March 16, 2006

All your answers should be in simplified form (i.e., no summations or $\cdots$).

1 Solve the following recurrence,

$$a_0 = 0, \quad a_n = 4a_{n-1} + 3^n \quad \text{for } n \geq 1.$$

2 The triangular number T_n is given by

$$T_n = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

As to the name triangular, imagine that you are making a triangle of oranges. On the first row you put n oranges, on the second row you put $n - 1$ oranges, on the third row $n - 2$ and so on, so that T_n is the number of oranges needed to form a triangle with sides of length n .

Solve the following recurrence (put your answer in the simplest possible form),

$$b_1 = 1, \quad b_n = b_{n-1} + T_n \quad \text{for } n \geq 1.$$

The b_i are the “tetrahedral” numbers, i.e., the number of oranges needed to form a tetrahedron where the base is a triangle with sides of length n , the next layer a triangle with sides of length $n - 1$ and so on.

(*Hint:* calculate the first few terms and look at binomial coefficients.)

3 Given that

$$c_0 = 5, \quad c_5 = 17, \quad c_n = 4c_{n-1} - 4c_{n-2} \quad \text{for all } n,$$

find the exact value of c_1 .

4 Let x and y be positive numbers and consider the following recursion,

$$d_1 = x, \quad d_2 = y, \quad d_{n+2} = \frac{1 + d_{n+1}}{d_n} \quad \text{for } n \geq 1.$$

Find $d_{527,619}$ in terms of x and y .

5 Solve the following recurrence

$$e_0 = 3, \quad e_1 = 1, \quad e_n = e_{n-1} + 6e_{n-2} + n \quad \text{for } n \geq 2.$$

(*Hint:* the “ $+n$ ” portion is what makes this hard. One way to deal with it is to suppose that $e_n = f_n + An + B$ and then choose good values for A and B so that the recursion for f_n is simple. This is similar to the method of undetermined coefficients in solving differential equations.)