

Example 1

Items: $\left(\frac{5}{5}\right), \left(\frac{8}{9}\right), \left(\frac{3}{4}\right)$

b	x_1	x_2	x_3	W	V	G
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	1	4	3	3
5	1	0	0	5	5	0
6	1	0	0	5	5	0
7	1	0	0	5	5	0
8	0	0	2	8	6	1
9	0	1	0	9	8	3
10	2	0	0	10	10	0
11	2	0	0	10	10	0
12	2	0	0	10	10	0
13	0	1	1	13	11	1
14	1	1	0	14	13	3
15	3	0	0	15	15	0

Example 2

Items: $\left(\frac{5}{5}\right), \left(\frac{7}{8}\right), \left(\frac{1}{2}\right)$

b	x_1	x_2	x_3	W	V	G
1	0	0	0	0	0	0
2	0	0	1	2	1	1
3	0	0	1	2	1	1
4	0	0	2	4	2	2
5	1	0	0	5	5	0
6	1	0	0	5	5	0
7	1	0	1	7	6	1
8	0	1	0	8	7	2
9	0	1	0	8	7	2
10	2	0	0	10	10	0
11	2	0	0	10	10	0
12	2	0	1	12	11	1
13	1	1	0	13	12	2
14	1	1	0	13	12	2
15	3	0	0	15	15	0

Example 3

Items: $\left(\frac{5}{5}\right), \left(\frac{6}{7}\right), \left(\frac{3}{4}\right)$

b	x_1	x_2	x_3	W	V	G
5	1	0	0	5	5	0
6	1	0	0	5	5	0
7	0	1	0	7	6	1
8	0	1	0	7	6	1
9	1	0	1	9	8	3
10	2	0	0	10	10	0
11	2	0	0	10	10	0
12	1	1	0	12	11	1
13	1	1	0	12	11	1
14	2	0	1	14	13	3
15	3	0	0	15	15	0
16	3	0	0	15	15	0
17	2	1	0	17	16	1
18	2	1	0	17	16	1
19	3	0	1	19	18	3

Weighted Interval Scheduling

- Each interval has a length and a value
- Select a subset of intervals so that the total value is maximum and no intervals are overlapping

Weighted Interval Scheduling

We sort the intervals by f_i :

$$f_1 \leq f_2 \leq f_3 \leq \cdots \leq f_n$$

$$\begin{array}{r} v_1 = 2 \\ \hline v_2 = 4 \\ \hline v_3 = 4 \\ \hline v_4 = 7 \\ \hline v_5 = 2 \\ \hline v_6 = 1 \\ \hline \end{array}$$

Weighted Interval Scheduling

- If v_6 is selected, then we must do optimally among all intervals not overlapping with v_6
- If v_5 is selected, then we must do optimally among all intervals not overlapping with v_5

Partitioning

- A book has 9 chapters.
- Goal: To bind the book into two volumes so that the bigger volume is not too heavy.
- The chapters have the following numbers of pages: 10, 20, 30, 40, 50, 60, 70, 80, 90.

E.g. Put chapters 1-6 into one volume and chapters 7-9 in the other.

Partitioning in General

- Partition a sequence $a_1, a_2, a_3, \dots, a_n$ into k subsequences
- Goal: Minimize the maximum sum of a subsequence

Define:

$$M[n, k] = \min_{i=1}^n \left(\max \left(M[i, k-1], \sum_{j=i+1}^n a_j \right) \right)$$

Partitioning Base Cases

- $M[1, k] = a_1, \forall k > 0$

If we have one chapter, we do not bind it into volumes

- $M[n, 1] = \sum_{i=1}^n a_i$

A single volume must consist of all pages.

Partitioning Example

- Chapters with weights 1, 2, 3, 4, 5, 6, 7, 8, 9
- k Volumes

$n \backslash k$	1	2	3
1	1	1	1
2	$1 + 2 \Rightarrow 3$	$1, 2 \Rightarrow 2$	$1, 2 \Rightarrow 2$
3	$1 + 2 + 3 \Rightarrow 6$	$1 + 2, 3 \Rightarrow 3$	$1, 2, 3 \Rightarrow 3$
4	$1 + \dots + 4 \Rightarrow 10$	$1 + 2 + 3, 4 \Rightarrow 6$	$1 + 2, 3, 4 \Rightarrow 4$
$\vdots$			