

CSE 20: Week 7 Homework Solutions

Douglas Turnbull

Department of Computer Science, UC San Diego
dturnbul@cs.ucsd.edu

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SF: Solutions to 1.15, 1.16, 1.19, 1.20, 2.4, 2.6, 2.9, 2.12 are found in the back of your textbook.

1. Prove using the element method that, for all subsets, P, Q, R of U , $Q - (P \cap R) = Q - (P \cap Q \cap R)$.

NOTE: To gain intuition, it might be helpful to try the proof using Venn diagrams or set algebra.

NOTE: When proving using the element method, we have that $LHS \subseteq RHS$ and $RHS \subseteq LHS$ in order to conclude $LHS = RHS$. (In our example the left-hand set (LHS) is $Q - (P \cap R)$ and the right-hand set (RHS) is $Q - (P \cap Q \cap R)$). Each subproof requires that we assume that x is an element of the subset and show that x is an element of the superset.

First we show that $Q - (P \cap R) \subseteq Q - (P \cap Q \cap R)$. Let $x \in Q - (P \cap R)$. Then $x \in Q$ and $x \notin P \cap R$. Then $x \notin (P \cap Q \cap R)$ since $P \cap Q \cap R$ is a subset of $P \cap R$. Then $x \in Q \cap (P \cap Q \cap R)^c$, or similarly by the definition of set difference, $x \in Q - (P \cap Q \cap R)$.

Next we show that $Q - (P \cap Q \cap R) \subseteq Q - (P \cap R)$. Let $x \in Q - (P \cap Q \cap R)$. Then $x \in Q$ and $x \notin (P \cap Q \cap R)$. Then we know that $x \notin (P \cap R)$ because we know x is in Q but not in either P or R , (or not in both P and R). Therefore x cannot be in the intersection of P and R . Then $x \in Q \cap (P \cap R)^c$, or similarly $x \in Q - (P \cap R)$.

Since $Q - (P \cap Q \cap R) \subseteq Q - (P \cap R)$ and $Q - (P \cap R) \subseteq Q - (P \cap Q \cap R)$,
 $Q - (P \cap R) = Q - (P \cap Q \cap R)$