

CSE 20: Week 3 Homework Solutions

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LO: Solutions to 2.8, 2.14, 2.18, 2.19 are found in the back of your textbook. Review questions 7 and 9 are found at the end of the LO section.

2.19 - The counter-example for (b), and (c) are slightly reversed. Let $D = \mathbb{Z}$, let $P(x)$ be “ x is even” and let $Q(x)$ be “ x is odd”.

For (b), the second statement $(\exists x \in \mathbb{Z}, "x \text{ is even}") \wedge (\exists x \in \mathbb{Z}, "x \text{ is odd}")$ is true, but the first statement $\exists x \in \mathbb{Z}, "x \text{ is odd"} \wedge "x \text{ is even}"$ is false.

NT: Solutions to NT {1.3, 1.8, 1.11, 1.17} are in the book.

Notes:¹ A good way to prove “if and only if” statements is to prove the “if” part separately from the “only if” part. Let’s look at 1.3(a) for example.

Prove: The product of two integers is even if and only if at least one of them is even.

$\Rightarrow$ (the “if” part): Assume the product of two integers $x \cdot y$ is even (this is actually a given for the “if” part). So $x \cdot y = 0 \pmod{2}$. For the sake of contradiction, assume both numbers are odd. That is, $x = 1 \pmod{2}$ and $y = 1 \pmod{2}$. Then $x \cdot y = 1 \pmod{2}$, a contradiction. Therefore, at least one of x, y must be even.

$\Leftarrow$ (the “only if” part): You no longer assume that the product xy is even. That is what you must show now. However, you are now given that at least one of the integers is even. Without loss of generality say x is even (otherwise, relabel the two). So $x = 0 \pmod{2}$. Well, 0 times anything is $0 \pmod{2}$. Therefore, $xy = 0 \pmod{2}$, meaning the product is even.

Solow: A solutions to 5.7 is found on the textbook website - www.wiley.com/college/solow/

Solow 5.2 To show that $S \subseteq T$, we must show that for all elements $(x, y) \in S$, $(x, y) \in T$. This we pick any generic element $(x', y') \in S$ and show that this specific element $(x', y') \in T$. This correct answer is C .

¹Solution prepared by Jan Voung for CSE 20 - Fall 2005

Solow 5.14 Prove that $\forall a, b \in \mathcal{Q}$ in which one is nonzero, $a^2 + ab + b^2 > 0$.

$$\begin{aligned} a^2 + ab + b^2 &\geq \frac{a^2}{2} + ab + \frac{b^2}{2} \\ &= \left(\frac{a}{\sqrt{2}} + \frac{b}{\sqrt{2}}\right)^2 \\ &> 0 \end{aligned}$$

Line 1 follow from the fact that $a^2 \geq \frac{a^2}{2}$ and $b^2 \geq \frac{b^2}{2}$ since $a^2 \geq 0$ and $b^2 \geq 0$. Line 2 is just factorization of the result from Line 1. Line 3 follow from the fact that $n^2 > 0$ when n in a nonzero real number.