

CSE 20: Week 2 Homework Solutions

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BF: Solutions to 2.9, 2.10, 2.13, 2.15, and 2.16 are found in the back of your textbook.

LO: Solutions to 1.7, 1.10, 1.16 and 1.23 are found in the back of your textbook

Solow: Solutions to 3.7, 3.15, 4.6, 4.11, 9.3, 9.7 and 9.17 are found in the back of your textbook or the website for the textbook (www.wiley.com/college/solow/ .)

Solow 3.11: Prove that if a and b are even integers, then $(a + b)^2$ is an even integer.

Def: n is an even integer if and only if there exists an integer k such that $n = 2(k)$.

Proof:

- Let a and b be even integers.
- Then there exists an integers i and j such that $a = 2i$ and $b = 2j$.
- The number $(a + b)^2$ can be written as

$$(a + b)^2 = (2i + 2j)^2 \quad (1)$$

$$= 4i^2 + 8ij + 4j^2 \quad (2)$$

$$= 2(2i^2 + 4ij + 2j^2) \quad (3)$$

- Since the product of integers is an integer, $2i^2$, $2j^2$ and $4ij$ are integers. Since the sum of integers is an integer, $(2i^2 + 4ij + 2j^2)$ is an integer.
- $(a + b)^2$ is an even integer since there exist an integer $(2i^2 + 4ij + 2j^2)$ such that $2(2i^2 + 4ij + 2j^2) = (a + b)^2$
- Therefore, if a and b are even integers, then $(a + b)^2$ is an even integer

Solow 9.8 Prove that there do not exist positive real numbers x and y with $x \neq y$ such that $x^3 - y^3 = 0$.

Proof: Note that $x > 0$ if and only if $x^3 > 0$ and $x < 0$ if and only if $x^3 < 0$. Assume that $x \neq y$. Since $x^3 = y^3$, $(x^3)^{1/3} = (y^3)^{1/3}$ or, simply $x = y$, which contradicts our assumption.

Note: Since $f(x) = x^3$ is a strictly monotonic function, there do not exist two distinct (positive or negative) real numbers ($x \neq y$) for which $f(x) = f(y)$. This would not be the case for a non-monotonic function such as $f(x) = x^2$. For this example, note that $f(-x) = f(x)$.