

CSE 20—Discrete Math

Winter, 2007

February 7, Day 9
Review of Midterm 1
Sets

Instructor: Neil Rhodes

1. 2 pts. How many negative 9-bit twos-complement numbers are there?

- (a) 512
- (b) 511
- (c) 256
- (d) 255
- (e) 128
- (f) 127
- (g) 0
- (h) 1

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2. 3 pts. How many distinct combinations of inputs are there for an n -input Boolean function?

- (a) 2^n
- (b) 2^{2^n}
- (c) n^2
- (d) n^{2^n}
- (e) n

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3. 3 pts. How many distinct n -input Boolean functions are there?

- (a) 2^n
- (b) 2^{2^n}
- (c) n^2
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4. 5 pts. Circle each of the following sets that are uncountable:

- (a) Mersenne primes
- (b) Real numbers
- (c) Irrational numbers
- (d) The set of students who will score 200% on this exam
- (e) The set of all subsets of rational numbers

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5. 10 pts. Use twos-complement 6-bit arithmetic to compute $-5 - 22$.
Show your work.

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6. 7 pts. Convert 57260563_8 to hexadecimal.

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7. 15 pts. Design a boolean function (using the operators $\wedge$, $\vee$, and $\sim$) equal to S as defined by the following truth table:

p	q	r	S
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

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8. 15 pts. Prove that if a, b , and c are integers for which $a|(b+c)$ and $a|b$, then $a|c$.

9. 20 pts. Given:

- predicates $I(x)$ ="x is wise" and $E(x,y)$ ="x knows y", and a
- domain $P = \{a, b, c\}$, and
- the truth set of $I(x)$ is $\{a\}$, and
- the truth set of $E(x,y)$ is $\{(a,a), (b,b), (c,c), (a,c), (a,b)\}$. [Remember that the truth set of a predicate is the set of elements in the domain that make the predicate true.]

Answer the following questions:

- (a) What is the truth value of $\forall x \in P \exists y \in P : I(y) \rightarrow E(x,x)$?

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- (b) What is the truth value of $\exists y \in P : \forall x \in P \sim E(x,y) \vee \sim I(x)$

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- (c) What is the opposite (in English) of "everyone who is wise knows themselves"?

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(d) Convert the following from predicate logic to English: $\exists x \in P \forall y \in P: E(x,y) \rightarrow I(y)$.

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(e) Convert the following from English to predicate logic: anyone who knows everybody is wise.

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10. 20 pts. Draw a combinatorial circuit containing only 1-input *not* and 2-input *or* gates that is equal to the following Boolean function (Note: make the circuit as simple as possible):

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11. 15 pts. Extra credit. Prove the statement if true, otherwise find a counterexample:

Given $x \neq 0$ is rational, if y is irrational, then the product xy is irrational.

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Definitions

Set: Contains zero or more *elements*, repetitions ignored, order doesn't matter

- Example with no elements
 - $\{\}$ (alternative syntax: $\emptyset$)
- Example with three elements
 - $\{\text{john, sally, harry}\} = \{\text{harry, sally, john}\} = \{\text{harry sally, harry, john}\}$
- Example with infinite elements
 - $\{x: x \text{ is a positive integer}\}$
- What about:
 - $\{\emptyset\}$

Ordered set: Contains zero or more elements, repetitions *not* allowed, order *does* matter (not often used)

- $(\text{john, sally, harry}) \neq (\text{sally, harry, john})$

Ordered list: Contains zero or more elements, repetitions allowed, order *does* matter (used often)

- Ordered 4-tuple
 - $(\text{john, sally, harry, sally})$
- Ordered pair
 - (x, y)
- Ordered triple
 - (x, y, z)

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Set notation

$x \in S$

- x is an element of S

$S \subseteq T$

- S is a subset of T : $\forall x, x \in S \rightarrow x \in T$

$S \subset T$

- S is a proper subset of T : $S \subseteq T \wedge (\exists x: x \in T \wedge x \notin S)$

$S = T$

- S is equal to T : $S \subseteq T$ and $T \subseteq S$

$|S|$

- The cardinality (size) of set S : the number of elements of S

k -set

- A set with cardinality k

$\{x: x > 1\}$ (or $\{x \mid x > 1\}$)

- Defines the set by stating the properties shared by the elements

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Set Operations

$S \cup T$

- Union: $\{x: x \in S \vee x \in T\}$

$S \cap T$

- Intersection: $\{x: x \in S \wedge x \in T\}$

$S - T$ (or $S \setminus T$)

- Difference: $\{x: x \in S \wedge x \notin T\}$

Often we have some universe U , in mind

S^c (or $\sim S$)

- Complement of S : $U - S$

$S \oplus T$

- Symmetric difference: $(S - T) \cup (T - S)$

$S \times T$

- (Cartesian) product: $\{(x, y): x \in S \wedge y \in T\}$

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Venn Diagrams

Rectangle is universe

Circle (or ellipse) for each set

Overlap represents intersection

Example:

- $A \subseteq B$

- $A \cap B$

- $A \cap B^c$

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Proof Using Venn Diagrams

Warning: has limitations

Proving LHS=RHS (or LHS≠RHS)

- Create general Venn diagram for given sets
- (optional) Number each region
 - (Given k sets, how many regions?)
- Shade in regions for LHS
- Shade in regions for RHS
- Compare

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Example Proof

Prove $(A \cap B)^c = A^c \cup B^c$

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Proof Using Elements

To prove LHS=RHS

- Prove $LHS \subseteq RHS$
- Prove $RHS \subseteq LHS$

To prove $LHS \subseteq RHS$

- If $x \in LHS$, then ...
- Thus, $x \in RHS$
- This shows that $LHS \subseteq RHS$

Example: prove $(A \cap B)^c = A^c \cup B^c$

- If $x \in (A \cap B)^c$, then $x \notin (A \cap B)$
- Thus, $x \notin A$ or $x \notin B$
- So, $x \in A^c$ or $x \in B^c$
- Hence, $x \in A^c \cup B^c$
- This shows that $(A \cap B)^c \subseteq A^c \cup B^c$

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Algebraic Rules for Sets

Associative	$A \cup (B \cap C) = (A \cup B) \cap C$	$A \cap (B \cup C) = (A \cap B) \cup C$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
DeMorgan's	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
Idempotent	$A \cup A = A$	$A \cap A = A$
Double Negation	$(A^c)^c = A$	
Absorption	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Bound	$A \cup \emptyset = A$	$A \cap \emptyset = \emptyset$
Negation	$A \cup A^c = U$	$A \cap A^c = \emptyset$

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Proof Using Algebraic Rules

$$A = (B^c \cup A) \cap (A \cup B)$$

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Tabular Method

$$A = (B^c \cup A) \cap (A \cup B)$$

Very similar to truth tables

A	B	B^c	$B^c \cup A$	$A \cup B$	$(B^c \cup A) \cap (A \cup B)$
€	€				
€	€				
€	€				
€	€				

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Tabular Method vs. Venn Diagram

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